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CONSTANT-K COMPLEMENTARY FILTERS 159

by

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INTRODUCTION

A goal in fan-out filter design is construction of a single input, denumerably infinite output spectrum analyzer that exhibits the individual terms of the input Fourier series. This entails a fan-out filter network that has a resistive input driving point impedance or constant interactance, and each fan-out that has a narrow band-pass filter transfer function and predictable linear phase shift.

Though the spectrum analyzer is the final goal, this thesis will consider a simpler problem which is a one input-two output filter designated as the complementary filter by Zobel (8). Early research will be discussed briefly with special attention to Fritzemeyer's (1) conceptual simplifications such as interactance and impedance elision. Constant-k complementary filter designs will be presented. Improvement of filter interactance and transfer function filter characteristics will be accomplished by the aidentity algorithm and impedance elision. Fritzemeyer investigated Newton complementary filters and these are not considered in this thesis. A large number of appendices contain proofs and digital computer programs necessary for the development of constant-k complementary filters.

PREVIOUS WORK

General

O. J. Zobel (8) was concerned with improving the driving

point impedance (DPI) of a constant-k filter and a constant-k complementary filter. Zobel added series and shunt elements in front of the constant-k filter and then empirically determined the value of these elements. In his constant-k complementary filter the shunt annulling branches were eliminated, similar to impedance elision, to obtain the best DPI results. No investigation of the voltage transfer function (VTF) was made.

Bode (5) was primarily interested in improving the DPI of a constant-k filter by adding a conductance controlling network and a susceptance annulling network. Later Bode (4) used these methods on constant-k complementary filters and a type of impedance elision with pole-zero analysis to improve the DPI.

Guillemin (10) investigated complementary and potentially complementary constant-k filters with added series elements that had properly chosen coefficients. The resulting filter was essentially that which Zobel and Bode had suggested.

Norton (11) working with complementary filters determined what the form of the equivalent DPI needs to be in order to exhibit a constant interactance and then attempted to synthesize the complementary filters which had this DPI. The calculations necessary for this synthesis are rather complex as they involve the solution of several simultaneous nonlinear equations.

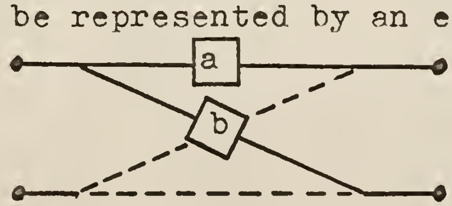
Szentirmai (7) suggested DPI improvement by adding resistive pads (networks) in each fan-out of a multiple output filter. This had the disadvantage of high losses due to the pads, although it does make each individual fan-out insensitive to each of the other fan-out networks.

Rowlands (6) suggested deletion of components nearest the paralleled terminals of complementary fan-in filters. This is impedance elision alright but no justification is given.

Fritzemeyer (1) used these early efforts and King's (2) approximate identity to investigate design procedures for obtaining the most constant DPI in constant-k and complementary filters. He suggested a different type of impedance elision than did Bode or Zobel and his suggested aidentity algorithm was different and realizable when compared with Norton's method. However none of these men were concerned with the VTF of their resultant filters and it is felt that this cannot be ignored. Therefore this thesis was initiated and completed with favorable results utilizing the suggestions of the referenced authors.

DEFINITIONS AND DESCRIPTIONS

<u>Lattice Representation of Symmetric Networks.</u> Any symmetric network may be represented by an equivalent lattice such as:



or the couple, $\{a;b\}$, where $a=f_1(s)$ and $b=f_2(s)$. $\{a;b\}$ will be employed in place of the lattice in textual material for compactness.

Characteristic Impedance. Any symmetric network, $\{a;b\}$, has a characteristic impedance, Z_o, equal to $\sqrt{ab}$.

Constant-K Low-Pass Filter. A symmetric low-pass filter can have a characteristic impedance which is an approximation of a desired characteristic impedance, k ohms. For instance the prototype low-pass filter, $\{Z_1/2;\ (Z_1+4Z_2)/2\}$, has a characteristic impedance, $k\sqrt{1+(Z_1/4Z_2)}$ where $k=\sqrt{Z_1Z_2}$, and a cut-off frequency, $w_c=2/\sqrt{L_1C_2}$ if $Z_1=L_1s$ and $Z_2=1/C_2s$. See Fig. 1.

Normalized Low-Pass Filter. A network with circuit components adjusted so that the characteristic impedance is one ohm and the cut-off frequency is one radian per second and is designated by the couple $(Z_o,\ w_c)=(1,1)$. This normalization permits computational ease without lessening generality. It is always possible to retrieve specified $(Z_o,\ w_c)$ by an impedance level change and a time scale change (i.e., s is replaced by τs).

Constant-1 Low-Pass Filter. A normalized symmetric low-pass filter with $k=1$ and $w_c=1$ rad./sec. These normalizations require $L_1=2$ henrys and $C_2=2$ farads for the prototype filter of Fig. 1 which results in the Constant-1 filter, $\{s;(s^2+1)/s\}$, of Fig. 2. Note that the actual characteristic impedance is $\sqrt{1+s^2}$ and this is an approximation to $k=1$.

Zobel's M-Derivation. Given $\{a;b\}$ and a constant m

such that $o<m<1$, then the network, $\{ma;\ b/m\}$, is a network m-derived from the given network. The two networks have the same characteristic impedance and the order of cascading can be ignored. Fig. 3 shows m-derivation with both a T-network and its lattice equivalent.

Network Transposition. The transpose of a four-terminal network is accomplished by transposing the network's right and left terminal pairs. The transposed network has a transfer matrix with the a_{11} and a_{22} elements interchanged. If network transposition leaves the network unchanged, then the given network is symmetric.

Zobel Filter. A symmetric network obtained by the process of m-deriving a T-network prototype, $\{s;(s^2+1)/s\}$; bisecting the resulting $\{ms;(s^2+1)/ms\}$; and inserting the prototype between its bisected and transposed m-derivation. Fig. 4 illustrates a Zobel filter normalized to (1,1).

Degenerate Zobel Filter. A symmetric network obtained by taking a Zobel filter and replacing the constant-1 filter with the identity network. Fig. 5 illustrates a degenerate Zobel filter.

Bisected Zobel Filter. A symmetric network obtained by taking a Zobel filter and replacing the constant-1 filter with a m-derived π-network. Fig. 6 illustrates a bisected Zobel filter.

Aidentity. A ratio of polynomials in s whose numerator and denominator are arranged in increasing powers of s and whose ratio approximates the constant, 1.

Aidentity Driving Point Impedance (ADPI). A driving point impedance (DPI) with the aidentity property. Such a DPI is both a positive real function (prf) and an aidentity.

Aidentity Order. In an aidentity if the first p successive pairs of coefficients of s^k, $o \le k \le p-1$, in the numerator and denominator are equal, the aidentity order is p.

Complementary Filter. A low-pass filter's input terminals connected in parallel with a high-pass filter's input terminals results in a complementary filter. The cut-off frequencies of the two filters are the same.

ASSERTIONS

Assertion 1. Two different symmetric networks can have the same characteristic impedance.

Proof: Given $\{a;b\}$ and $\{ma;b/m\}$. Both have the same Z_o and yet they have different circuit components and transfer characteristics.

Specific examples in the prototype low-pass filter context are given in Figs. 7, 8, 9, and 10. It is observed that the bisected process of Fig. 10 yields a different example network having the same Z_o as the three previous example networks.

Assertion 2. Whereas the m-derived networks of Assertion 1 have a Z_o independent of m, the ADPI's of the m-derived networks in Assertions 3, 4, and 5 are dependent on m.

Assertion 3. The Zobel process or m-derived half end termination process improves the characteristic impedance of the networks of Assertion 1 after m is chosen to be 0.707. This is shown in Fig. 11.

Assertion 4. The ADPI's of the constant-1, m-derived, composites and bisected Zobel filters and loads are different, but each has the same aidentity order. In contrast to Assertion 2, the ADPI's orders are not dependent on m for these cases shown in Figs. 12, 13, 14, 15, and 16.

Assertion 5. The aidentity order of the Zobel filter and the degenerate Zobel filter is greater by two than the aidentity order of the Assertion 4 filters; and in contrast to Assertion 4, the ADPI's orders are dependent on m=0.707 for the cases of Fig. 17 and Fig. 18.

Conclusions

The Zobel process effects can be summarized as follows:

a. When the characteristic impedance, Z_o, is dependent on m, there is a resulting improvement of Z_o and ADPI order when m is chosen to be 0.707.

b. When the characteristic impedance, Z_o, is independent of m, there is no improvement of Z_o or ADPI order from any realizable choice of m.

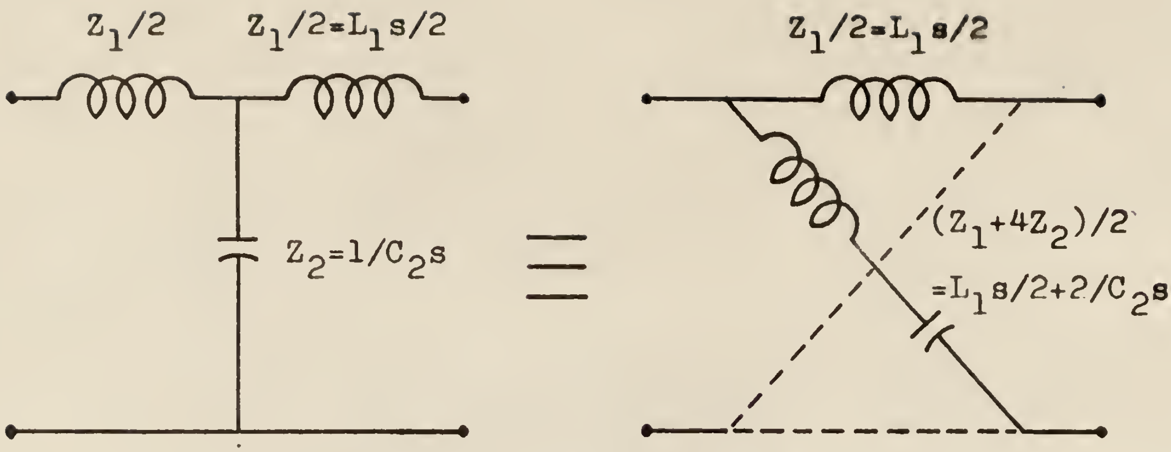


Fig. 1. The prototype low-pass filter and its lattice equivalent.

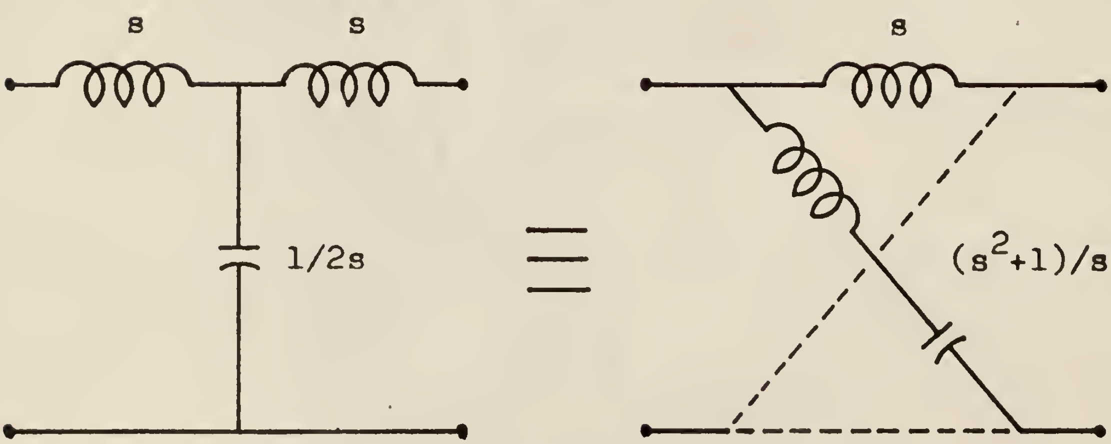


Fig. 2. The constant-1 low-pass filter and its lattice equivalent, $\{s;(s^2+1)/s\}$.

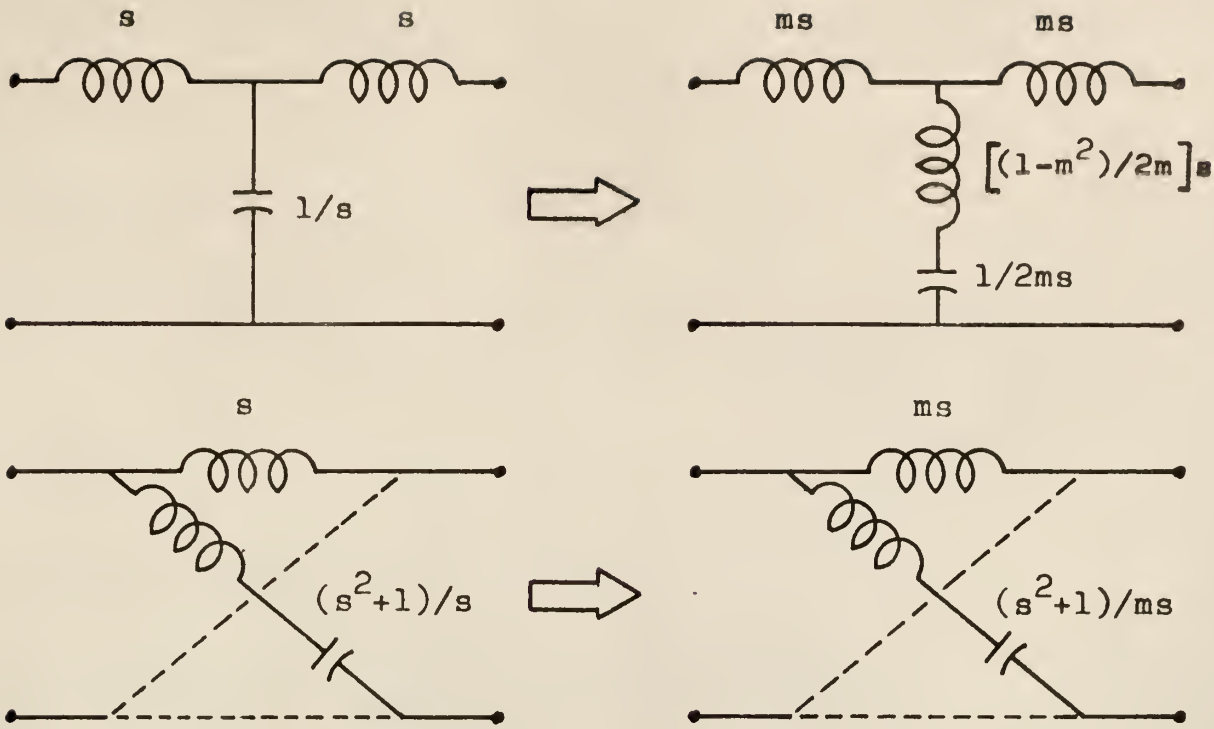


Fig. 3. M-derivation of a T-network and its equivalent lattice.

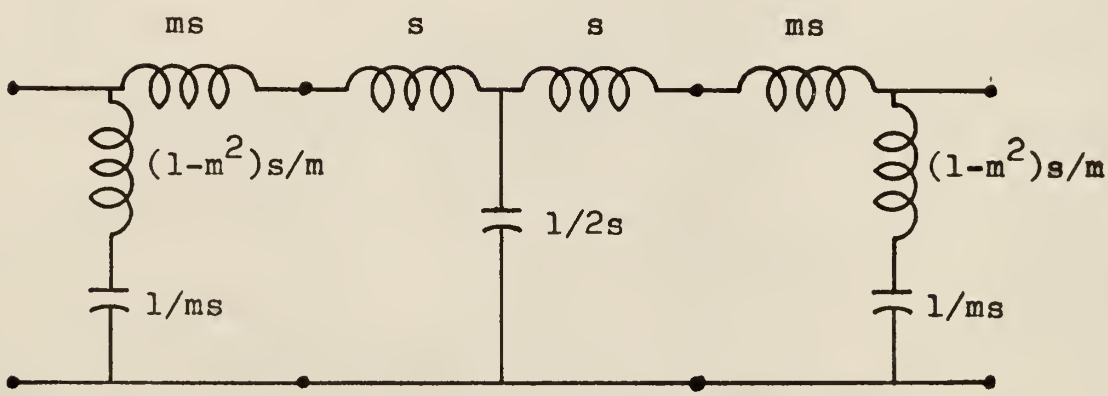


Fig. 4. Normalized (1,1) Zobel filter.

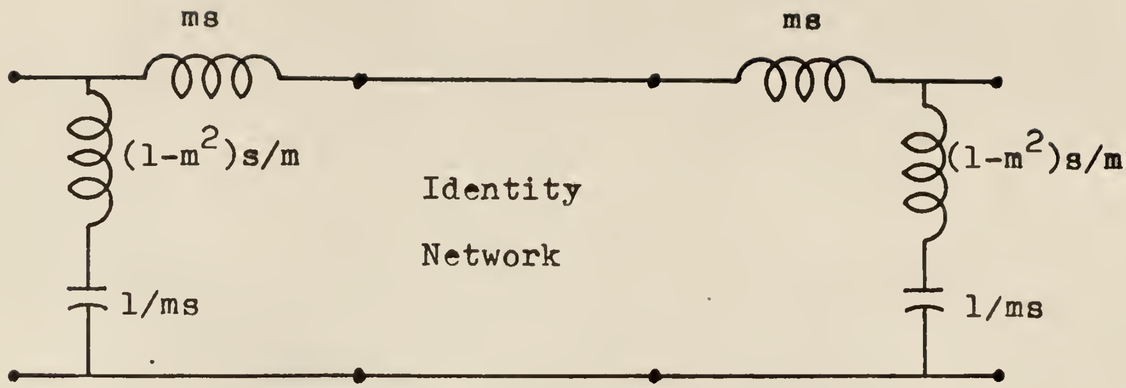


Fig. 5. Degenerate Zobel filter.

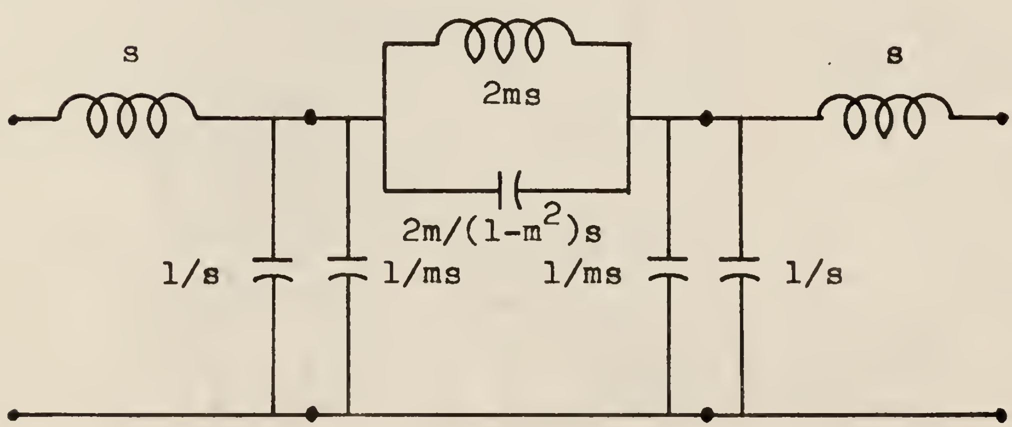


Fig. 6. Bisected Zobel filter.

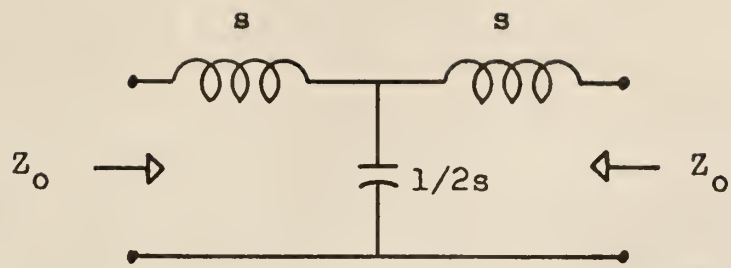


Fig. 7. Constant-l filter, $Z_o=\sqrt{1+s^2}$.

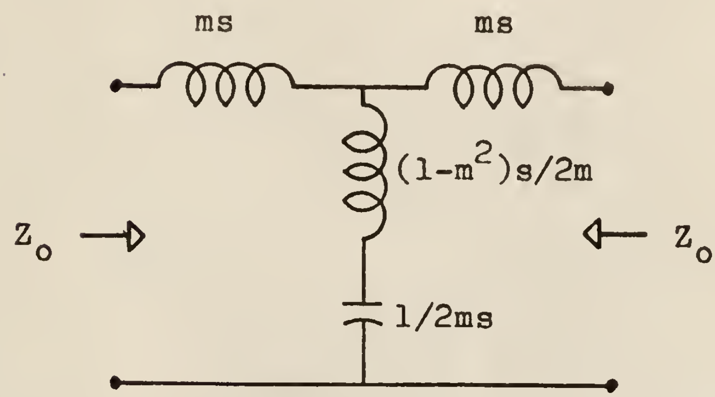


Fig. 8. M-derived filter, $Z_o=\sqrt{1+s^2}$.

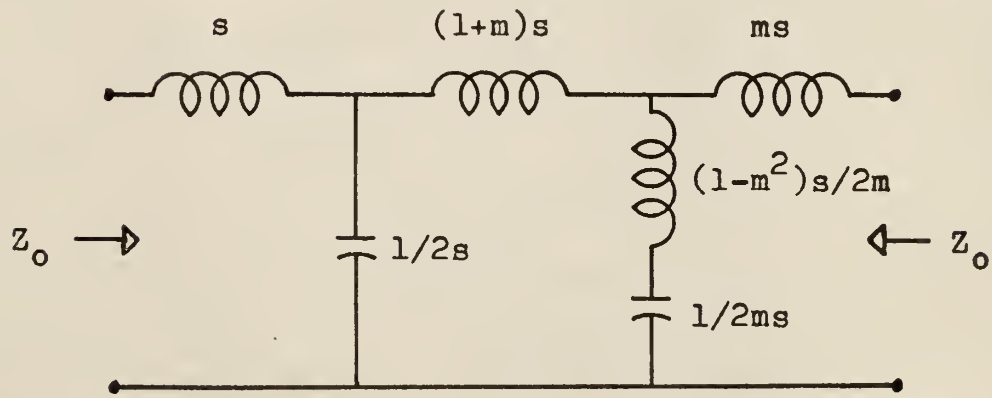


Fig. 9. Composite filter, $Z_o=\sqrt{1+s^2}$.

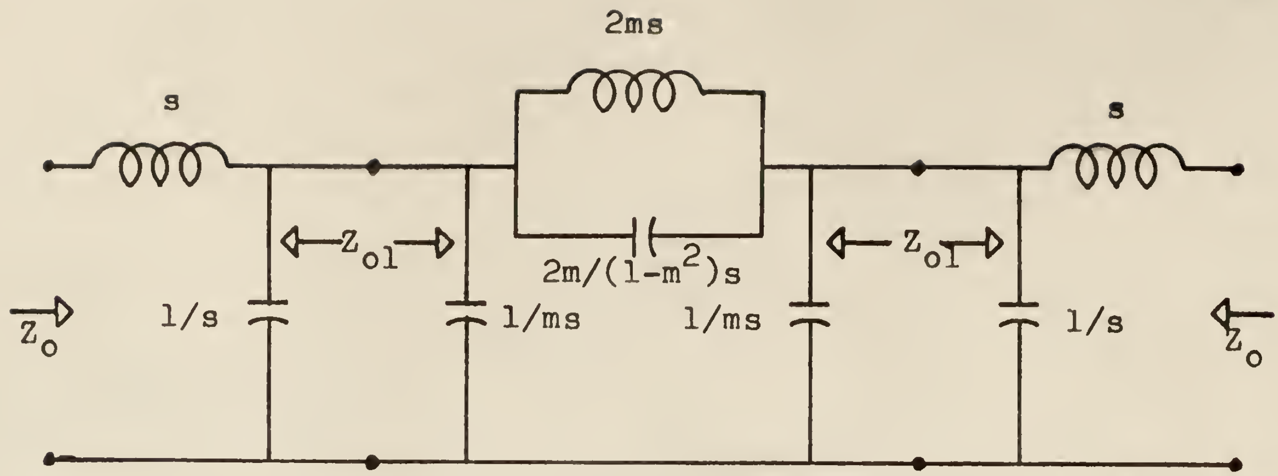
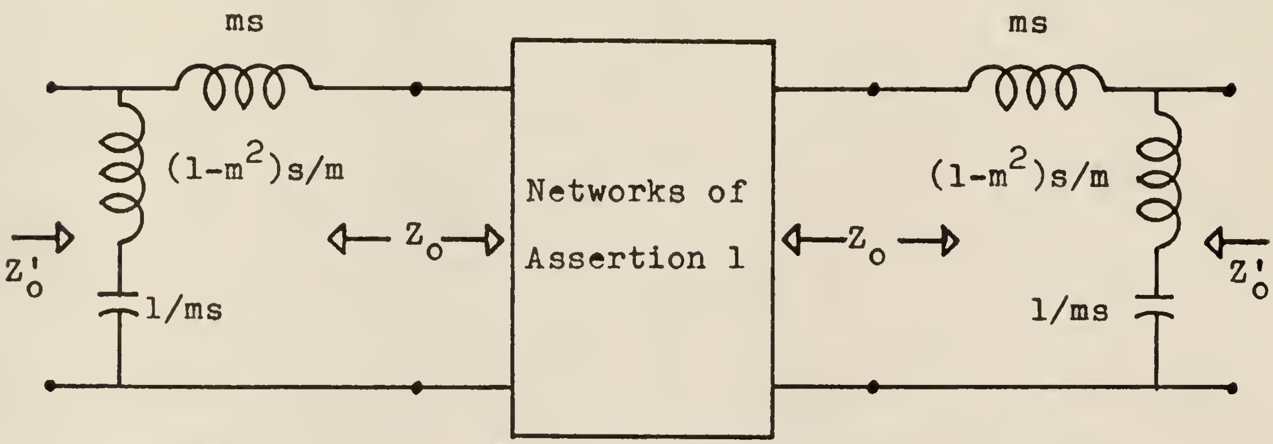


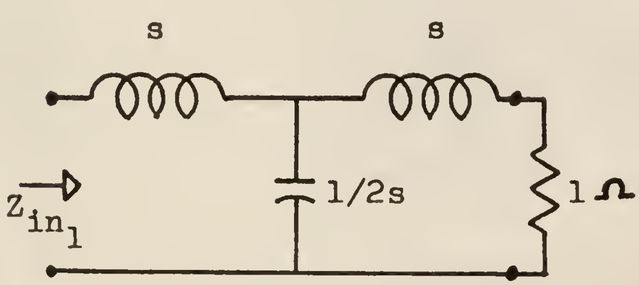
Fig. 10. Bisected Zobel filter, $Z_o=\sqrt{1+s^2}$ and $Z_{o1}=\sqrt{1/(1+s^2)}$.



$Z_o=\sqrt{1+s^2}$ and $Z_o'=\sqrt{\left[1+2(1-m^2)s^2+(1-m^2)^2s^4\right]/(1+s^2)}$

If m=0.707, then $Z_o'=\sqrt{1+\left[s^4/4(1+s^2)\right]}$

Fig. 11. Zobel's method of Z_o improvement.



$$Z_{in_1}=\frac{A_o+A_1s+A_2s^2+A_3s^3}{B_o+B_1s+B_2s^2}$$

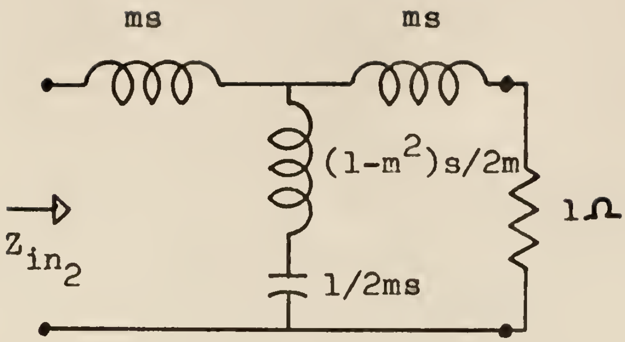
$A_o=1$ $B_o=1$

$A_1=2$ $B_1=2$

$A_2=2$ $B_2=2$

$A_3=2$

Fig. 12. Constant-1 filter whose input impedance is a third order ADPI.



$$Z_{in_2}=\frac{C_o+C_1s+C_2s^2+C_3s^3}{D_o+D_1s+D_2s^2}$$

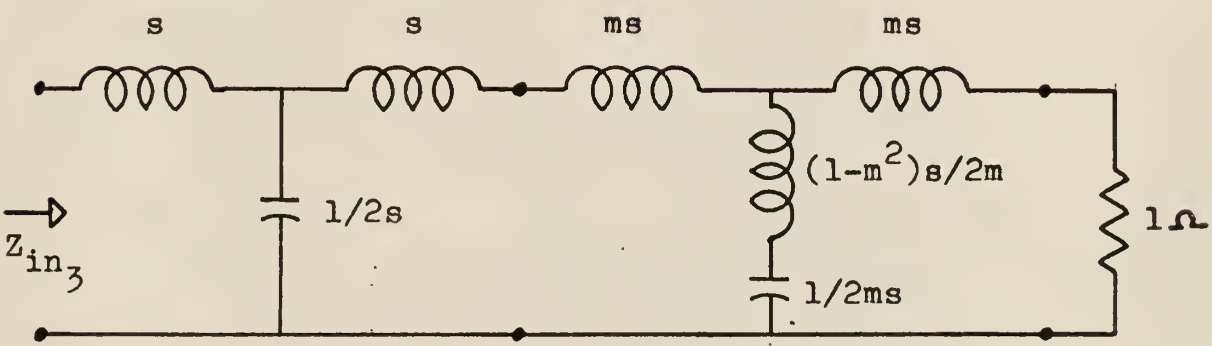
$C_o=1$ $D_o=1$

$C_1=2m$ $D_1=2m$

$C_2=1+m^2$ $D_2=1+m^2$

$C_3=2m$

Fig. 13. M-derived filter whose input impedance is a third order ADPI.



$$Z_{in_3}=\frac{E_o+E_1s+E_2s^2+E_3s^3+E_4s^4+E_5s^5}{F_o+F_1s+F_2s^2+F_3s^3+F_4s^4}$$

$E_o = 1$ $F_o = 1$

$E_1 = 2(1+m)$ $F_1 = 2(1+m)$

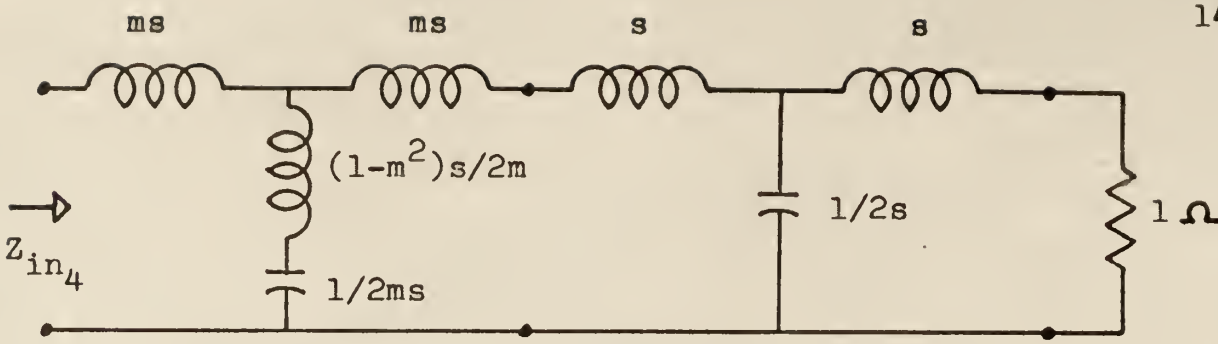
$E_2 = (3+m)(1+m)$ $F_2 = (3+m)(1+m)$

$E_3 = 2(2+m)(1+m)$ $F_3 = 2(1+m)^2$

$E_4 = 2(1+m)^2$ $F_4 = 2(1+m)^2$

$E_5 = 2(1+m)^2$

Fig. 14. Composite filter whose input impedance is a third order ADPI.



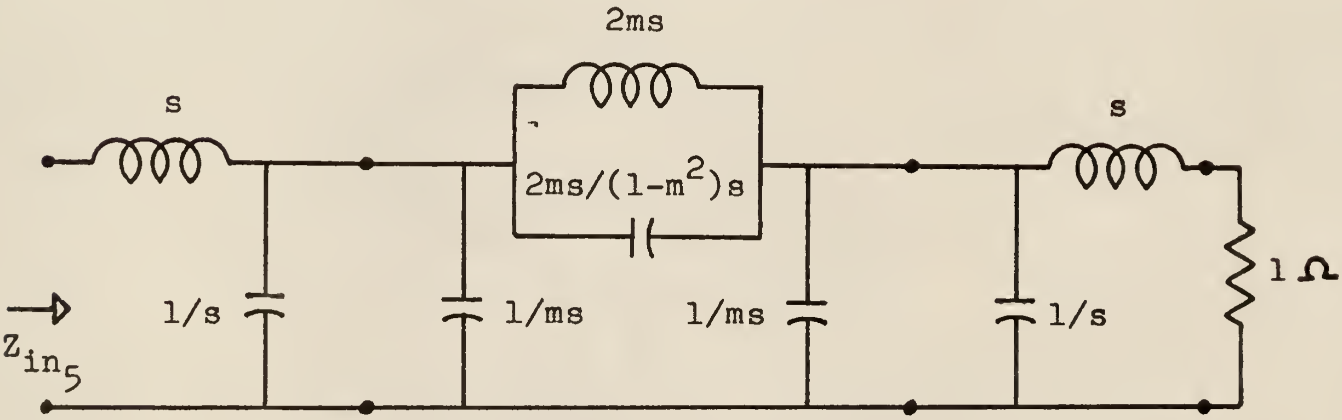
$$Z_{in_4}=\frac{G_o+G_1s+G_2s^2+G_3s^3+G_4s^4+G_5s^5}{H_o+H_1s+H_2s^2+H_3s^3+H_4s^4}$$

$G_o=1$ $\quad G_3=2(2+m)(1+m)$ $\quad H_o=1$ $\quad H_3=2(1+m)^2$

$G_1=2(1+m)$ $\quad G_4=2(1+m)^2$ $\quad H_1=2(1+m)$ $\quad H_4=2(1+m)^2$

$G_2=(3+m)(1+m)$ $\quad G_5=2(1+m)^2$ $\quad H_2=(3+m)(1+m)$

Fig. 15. Composite filter whose input impedance is a third order ADPI.



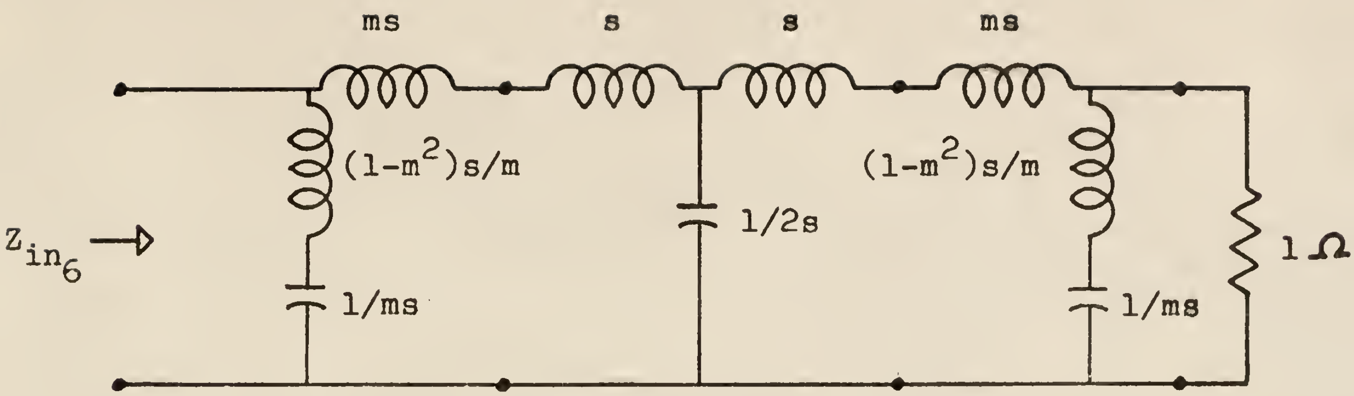
$$Z_{in_5}=\frac{I_o+I_1s+I_2s^2+I_3s^3+I_4s^4+I_5s^5}{J_o+J_1s+J_2s^2+J_3s^3+J_4s^4}$$

$I_o=1$ $\quad I_3=2(2+m)(1+m)$ $\quad J_o=1$ $\quad J_3=2(1+m)^2$

$I_1=2(1+m)$ $\quad I_4=2(1+m)^2$ $\quad J_1=2(1+m)$ $\quad J_4=2(1+m)^2$

$I_2=(3+m)(1+m)$ $\quad I_5=2(1+m)^2$ $\quad J_2=(3+m)(1+m)$

Fig. 16. Bisected Zobel filter whose input impedance is a third order ADPI.

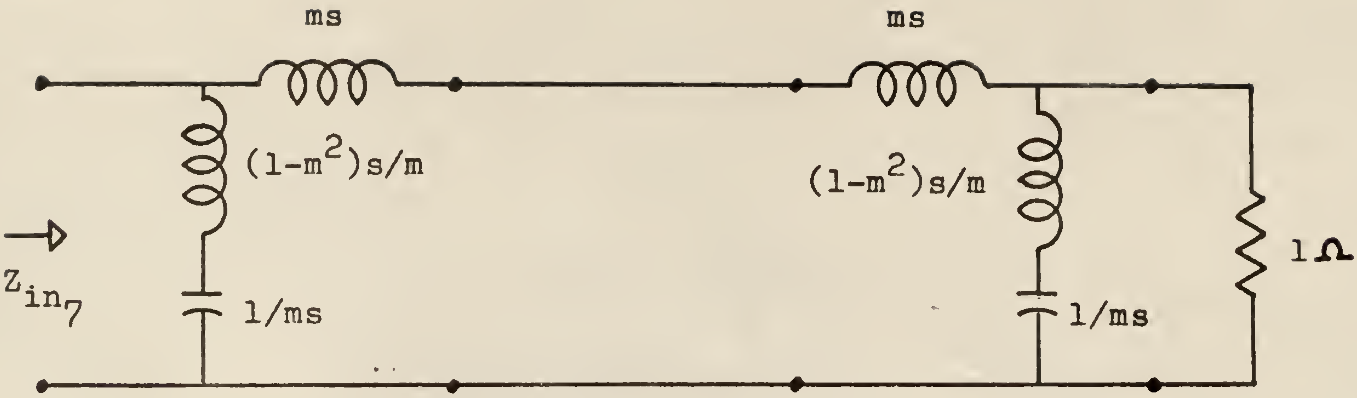


$$Z_{in_6} = \frac{K_o+K_1s+K_2s^2+K_3s^3+K_4s^4+K_5s^5+K_6s^6+K_7s^7}{L_o+L_1s+L_2s^2+L_3s^3+L_4s^4+L_5s^5+L_6s^6}$$

$K_o = 1$	$L_o = 1$
$K_1 = 2(1+m)$	$L_1 = 2(1+m)$
$K_2 = 4(1+m)$	$L_2 = 4(1+m)$
$K_3 = 2(1+m)^2(3-2m)$	$L_3 = 2(1+m)(2+m)$
$K_4 = (1+m)^2\ 2+(3+m)(1-m)$	$L_4 = (1+m)^2(5-2m-m^2)$
$K_5 = 2(1+m)^3(1-m)(3-m)$	$L_5 = 2(1+m)^2$
$K_6 = 2(1+m)^3(1-m)$	$L_6 = 2(1+m)^3(1-m)$
$K_7 = 2(1+m)^4(1-m)^2$	

If $m=0.707$, then $K_o=L_o$, $K_1=L_1$, $K_2=L_2$, $K_3=L_3$, and $K_4=L_4$.

Fig. 17. Zobel filter whose input impedance is a fifth order ADPI.



$$Z_{in_7} = \frac{M_o+M_1 s+M_2 s^2+M_3 s^3+M_4 s^4+M_5 s^5}{N_o+N_1 s+N_2 s^2+N_3 s^3+N_4 s^4}$$

$M_o = 1$	$N_o = 1$
$M_1 = 2m$	$N_1 = 2m$
$M_2 = 2$	$N_2 = 2$
$M_3 = 4m(1+m)(1-m)$	$N_3 = 2m$
$M_4 = (1+m)(1-m)(1+m^2)$	$N_4 = (1+m)(1-m)(1+m^2)$
$M_5 = 2m(1+m)^2(1-m)^2$	

If $m=0.707$, then $M_o=N_o$, $M_1=N_1$, $M_2=N_2$, $M_3=N_3$, and $M_4=N_4$.

Fig. 18. Degenerate Zobel filter whose input impedance is a fifth order ADPI.

CONSTANT-K COMPLEMENTARY FILTERS

Procedure

In the previous section it was shown that the Zobel process improved the characteristic impedance and ADPI of low-pass prototype filters. Now we shall compare the frequency response characteristics of the constant-1 filter, Zobel filter (with m having two different values), constant-1 complementary filters (modified by aidentity algorithm) and complementary Zobel filters (modified by impedance elision and the aidentity algorithm). The filter characteristics of interest are interactance, ADPI, and voltage transfer function (VTF). Interactance versus frequency is a unique filter figure of merit first presented by Fritzemeyer (1). Interactance is an effective filter design substitute for characteristic impedance. The interactance frequency response was calculated and tabulated on the IBM 1620 digital computer. A review of the interactance, the computer program, and the tabulated input and output data are given in Appendix C and Appendix D. The ADPI versus frequency was also computed on the 1620 and this program and results are given in Appendix E. Impedance elision is presented in Appendix A and the aidentity algorithm is presented in Appendix B. All of these filters have the ladder configuration and a computer program was written to determine the coefficients of each filter's low-pass VTF. This is given in Appendix F with the computer program and the resultant data given in Appendix G. Appendix

H shows the computer program and results for computing the low-pass VTF frequency response.

Constant-1 Filter

Considering the low-pass constant-1 filter of Fig. 19 first will give a basis of comparison for the various improvements of the filter characteristics of the Zobel filters. The ADPI of equation (1) for the constant-1 filter has an aidentity order that can not be increased by the aidentity algorithm as is shown in Appendix B. Fig. 20 shows the interactance curve which increases rapidly between 0.3 rad/sec and 1.0 rad/sec.

$$Z_{in} = \frac{A_o+A_1s+A_2s^2+A_3s^3}{B_o+B_1s+B_2s^2} \qquad (1)$$

where

$$\begin{array}{ll} A_o = 1 & B_o = 1 \\ A_1 = 2 & B_1 = 2 \\ A_2 = 2 & B_2 = 2 \\ A_3 = 2 & \end{array} \qquad (2)$$

This indicates that the ADPI aidentity order has to be increased to improve the interactance. The interactance is verified by the ADPI curve. The low-pass VTF curve does not have a very sharp knee and the VTF phase shift curve is linear throughout the pass band as shown in Fig. 21.

Zobel Filter

Using the common value of 0.6 for m in the Zobel filter of Fig. 22, the ADPI of equation (3) for this filter has an aidentity order of one and has the characteristic responses of Fig. 23 and Fig. 24.

$$Z_{in} = \frac{C_o+C_1 s+C_2 s^2+C_3 s^3+C_4 s^4+C_5 s^5+C_6 s^6+C_7 s^7}{D_o+D_1 s+D_2 s^2+D_3 s^3+D_4 s^4+D_5 s^5+D_6 s^6} \qquad (3)$$

where

$$\begin{array}{llll} C_o = 1 & & D_o = 1 & \\ C_1 = 3.84 & & D_1 = 3.2 & \\ C_2 = 5.76 & & D_2 = 8.06 & \\ C_3 = 11.63 & & D_3 = 6.64 & (4) \\ C_4 = 6.32 & & D_4 = 10.81 & \\ C_5 = 9.1 & & D_5 = 3.07 & \\ C_6 = 1.95 & & D_6 = 3.26 & \\ C_7 = 2.07 & & & \end{array}$$

However when the aidentity algorithm is applied to the ADPI as in Assertion 5 on page 7, equation (5), which has an aidentity order of five, is obtained and m is 0.707.

$$Z_{in} = \frac{C_o+C_1 s+C_2 s^2+C_3 s^3+C_4 s^4+C_5 s^5+C_6 s^6+C_7 s^7}{D_o+D_1 s+D_2 s^2+D_3 s^3+D_4 s^4+D_5 s^5+D_6 s^6} \qquad (5)$$

where

$$
\begin{aligned}
C_o &= 1 & \qquad D_o &= 1 \\
C_1 &= 3.414 & D_1 &= 3.414 \\
C_2 &= 6.828 & D_2 &= 6.828 \\
C_3 &= 9.243 & D_3 &= 9.243 \\
C_4 &= 8.992 & D_4 &= 8.992 \\
C_5 &= 6.682 & D_5 &= 5.828 \\
C_6 &= 2.914 & D_6 &= 2.914 \\
C_7 &= 1.457 & &
\end{aligned}
\qquad (6)
$$

This results in a Zobel filter with constants as shown in Fig. 25 and characteristic responses of Fig. 26 and Fig. 27.

In comparing the characteristic responses for the two choices of m, it is obvious that the interactance for m=0.707 is much better during the pass-band than for m=0.6. This was expected because of the difference in the aidentity order of the two ADPI'S. It is difficult to make a conclusion about the interactance improvement of the Zobel filter over the constant-l filter because of the extreme variations near w_c for the Zobel filter.

It is worth noting that Fritzemeyer's (1) interactance curve for the m-derived filter, with m=0.6 and ADPI aidentity order of 3, is essentially the same as the interactance curve of Fig. 26 for a Zobel filter with m=0.707 and ADPI aidentity order of 5. The validity of Fritzemeyer's curve is questioned.

It should be noted that the low-pass VTF response for m=0.6 has a slightly sharper knee than the one for m=0.707. However both have much better low-pass VTF responses than

the constant-l filter and both are approximately 0.35 at w_c. The VTF phase shift curves for both values of m are not as linear as was the constant-l phase shift curve. The phase shift is rather linear from zero to about 0.75 rad/sec, then the curve becomes slightly nonlinear. Also the Zobel filter phase shift slope is steeper than the slope for the constant-l filter. The ADPI phase shift is improved considerably over the m=0.6 Zobel filter and the constant-l filter.

Complementary Filters

Having looked at the characteristics of the prototype low-pass constant-l and Zobel filters, we will now observe the characteristics of complementary filters when the aidentity algorithm and impedance elision are applied.

Type 1 Constant-l Complementary Filter

When two constant-l complementary filters are connected as shown in Fig. 28, the ADPI of equation (7) results. No aidentity algorithm is possible with this ADPI as all components have fixed values. The symmetry of numerator and denominator coefficients should be observed. This is expected because of the mathematical operations in determining the equivalent parallel impedance when high-pass and low-pass impedance functions are involved. Appendix I provides a further explanation of this coefficient symmetry. Inspection of the interactance curve of Fig. 29 reveals that the complementary nature of this filter will result in a symmetrical

curve about w_c.

$$Z_{cf} = \frac{E_o+E_1s+E_2s^2+E_3s^3+E_4s^4+E_5s^5+E_6s^6}{F_o+F_1s+F_2s^2+F_3s^3+F_4s^4+F_5s^5+F_6s^6} \tag{7}$$

where

$$\begin{aligned} E_o &= E_6 = 2 & F_o &= F_6 = 2 \\ E_1 &= E_5 = 6 & F_1 &= F_5 = 8 \\ E_2 &= E_4 = 10 & F_2 &= F_4 = 16 \\ E_3 &= 13 & F_3 &= 18 \end{aligned} \tag{8}$$

Note that although the interactance curve is not flat, it does not have the extreme variations about w_c that the constant-l filter and Zobel filter each have. Its VTF response of the low-pass fan-out is better than the constant-l filter, but not quite as good as the Zobel filter. It is approximately 0.3 at w_c and doesn't reach 0.1 until w=1.5 rad/sec. However its low-pass VTF phase shift curve is as linear, but with less slope, as the VTF phase shift of the Zobel filter. Both have the same type of nonlinearity at w_c. The ADPI phase shift response has deteriorated by adding the complementary high-pass filter in parallel with the constant-l filter.

Type 2 Constant-l Complementary Filter with Parameters C and C'

To improve the interactance of the complementary configuration, parameters C and C', were added to the reactances nearest the input terminals of each filter (Fig. 31) so

that the aidentity algorithm could be used on the ADPI of equation (9). Again as stated in the previous complementary filter, $G_o=G_6$, $G_1=G_5$, $G_2=G_4$, $H_o=H_6$, $H_1=H_5$, and $H_2=H_4$. The aidentity algorithm is next applied which set $G_1=H_1$ and $G_5=H_5$.

$$Z_{cf} = \frac{G_o+G_1s+G_2s^2+G_3s^3+G_4s^4+G_5s^5+G_6s^6}{H_o+H_1s+H_2s^2+H_3s^3+H_4s^4+H_5s^5+H_6s^6} \tag{9}$$

where

$$\begin{aligned}
G_o &= 2C' = G_6 = 2C \\
G_1 &= 2C'(2 + C) = G_5 = 2C(2 + C') \\
G_2 &= 1 + 3C' + 6CC' = G_4 = 1 + 3C + 6CC' \\
G_3 &= 2 + C + C' + 9CC' \\
H_o &= 2C' = H_6 = 2C \\
H_1 &= 2(1 + 3C') = H_5 = 2(1 + 3C) \\
H_2 &= 5 + 2C + 9C' = H_4 = 5 + 2C' + 9C \\
H_3 &= 6(1 + C + C')
\end{aligned} \tag{10}$$

These two simultaneous equations yield the results, $C=C'=1.618$ and a second order ADPI. This suggests that the dual components of the complementary filters should have the same coefficients. This observation will be used on later complementary configurations. Using this value for C and C', Z_{cf} has the coefficients of equation (11) and the frequency response characteristics for the complementary filter of Fig. 31 given in Fig. 32 and Fig. 33. By adding the parameters C and C' and using the aidentity algorithm on Z_{cf},

$$G_0 = G_6 = 3.24 \qquad H_0 = H_6 = 3.24$$
$$G_1 = G_5 = 11.7 \qquad H_1 = H_5 = 11.7 \quad (11)$$
$$G_2 = G_4 = 21.55 \qquad H_2 = H_4 = 22.8$$
$$G_3 = 28.79 \qquad H_3 = 25.42$$

the interactance of Fig. 32 results which during the larger part of the high and low pass-bands is much better than for Type 1. However the interactance response about w_c has deteriorated from Type 1. The VTF response of the low-pass fan-out has been improved from Type 1 low-pass as it is flat for more of the pass-band, then has a sharper knee, a 0.312 value at w_c, and a 0.1 value at w=1.25 rad/sec. The low-pass VTF response is similar to that of the Zobel filter with m=0.707. The low-pass VTF phase shift curve is nearly identical with that of Type 1, with the same non-linearity at w_c. The ADPI phase shift response during most of the pass-bands is much better than for Type 1 except for the larger variations about w_c. It does have the same general shape as the ADPI phase shift curve of the Zobel filter.

Type 3 Constant-1 Complementary Filter with Parameters C and C_1

Using the idea of Type 2 that dual components of the complementary filter should have equal coefficients, the parameters C and C_1 were added to the constant-1 complementary filter (Fig. 34) with the idea of using the aidentity algorithm to improve the interactance and ADPI. The ADPI of equation (12)

$$Z_{cf} = \frac{I_o+I_1s+I_2s^2+I_3s^3+I_4s^4+I_5s^5+I_6s^6}{J_o+J_1s+J_2s^2+J_3s^3+J_4s^4+J_5s^5+J_6s^6} \qquad (12)$$

$$\begin{aligned}
\text{where}\quad I_o &= I_6 = CC_1 &&= 3.732 \\
I_1 &= I_5 = CC_1(2 + C) &&= 13.93 \\
I_2 &= I_4 = 1 + C + CC_1(1 + C + CC_1) &&= 26.86 \\
I_3 &= 2(1 + C) + C^2(1 + 2C_1{}^2) &&= 36.33 \qquad (13) \\
J_o &= J_6 \doteq CC_1 &&= 3.732 \\
J_1 &= J_5 = C_1(1 + C + CC_1) &&= 13.93 \\
J_2 &= J_4 = 1 + C + 2C_1 + CC_1(1 + 2C_1) &&= 26.86 \\
J_3 &= 2\left[1 + C_1 + CC_1(1 + C_1)\right] &&= 29.86
\end{aligned}$$

results from the circuit of Fig. 34. Using the aidentity algorithm on the simultaneous equations, $I_o=J_o$, $I_1=J_1$, and $I_2=J_2$, yields $C=1.732$ and $C_1=2.155$. Comparing the resultant component coefficients of Type 3 with Type 2, indicates that the changes are rather small. This implies that the interactance may not be improved much by this approach, although the ADPI order is now three. Examination of this filter's frequency response characteristics given in Fig. 35 and Fig. 36 confirms the above suspicions. There is even a more extreme interactance variation about w_c for Type 3 than for Type 2 which agrees with the large difference between I_3 and J_3. The low-pass VTF response even has a more rounded knee than did Type 2, however it reaches 0.1 at w=1.15 rad/sec instead of w=1.25 rad/sec as did Type 2. The one bright spot is the improvement of linearity in the low-pass VTF

phase shift curve about w_c which is not present in Type 2. Along with the other deteriorating features, the ADPI phase shift curve has greater variations about w_c than did Type 2, although it has less phase shift during most of its pass-band than Type 2. It seems that by increasing the aidentity order by one there is only a small improvement in the phase shift response.

Type 4 Constant-1 Complementary Filter with Parameters C, C_1 and C_2

The results of Type 3 seem to imply that any further attempt at improving the ADPI and interactance by addition of a third parameter would be futile. However when the three component parameters were used as shown in Fig. 37, the ADPI of equation (14) resulted. The aidentity algorithm using $K_o=L_o$, $K_1=L_1$, $K_2=L_2$, and $K_3=L_3$ yields $C=1.5$, $C_1=1.33$, and $C_2=0.5$ which then results in an aidentity order of seven. This bonus in aidentity order, of course, comes from the property of coefficient symmetry of the ADPI. The frequency response characteristics of Fig. 38 and Fig. 39 reflect an ADPI of one for all frequencies and a constant interactance.

$$Z_{cf} = \frac{K_o+K_1s+K_2s^2+K_3s^3+K_4s^4+K_5s^5+K_6s^6}{L_o+L_1s+L_2s^2+L_3s^3+L_4s^4+L_5s^5+L_6s^6} \qquad (14)$$

where

$$
\begin{aligned}
K_o &= K_6 = CC_1C_2 && = 0.998 \\
K_1 &= K_5 = CC_1(1 + CC_2 + C_2{}^2) && = 3.99 \\
K_2 &= K_4 = C + C_2 + CC_1(C + C_2 + CC_1C_2) && = 7.98 \\
K_3 &= 1 + C^2 + C_2{}^2 + 2CC_2 + C^2C_1{}^2(1 + C_2{}^2) && = 9.95 \qquad (15) \\
L_o &= L_6 = CC_1C_2 && = 0.998 \\
L_1 &= L_5 = C_1C_2 + CC_1(1 + C_1C_2) && = 3.99 \\
L_2 &= L_4 = C + C_1 + C_2 + C_1C_2{}^2 + CC_1(C_1 \\
&\qquad + C_2 + C_1C_2) && = 7.98 \\
L_3 &= 2\left[1 + C_1C_2 + CC_1(1 + C_1C_2)\right] && = 9.95
\end{aligned}
$$

This is the ultimate in driving point impedance characteristics for most filters. The low-pass VTF response has been improved by the w_c value being 0.353 while the rest of the pass-band response remains very nearly the same as Type 3. This finding is in disagreement with what Fritzemeyer (1) stated would be the response. The ADPI phase shift is a straight line at zero degrees as is expected. The low-pass VTF phase shift has less nonlinearity about w_c than did Type 3 and its slope is also less, which gives it a very good linear phase shift response.

Type 5 Complementary Zobel Filter

As has been the procedure of the two previous sections, the Zobel filter configuration is used to improve the ADPI of the complementary filter. Using the network of Fig. 40 results in the ADPI of equation (16). The determination

$$Z_{cf} = \frac{\sum_{n=o}^{14} M_n s^n}{\sum_{n=o}^{14} N_n s^n} \tag{16}$$

where

$a = 1 + m$

$b = 1 - m$

$$\begin{aligned}
M_o &= M_{14} = 2a^4b^2 \\
M_1 &= M_{13} = 2a^3b(1+2a^2b) \\
M_2 &= M_{12} = 2a^3b\left[2(a-b) + 4a^2b\right] \\
M_3 &= M_{11} = 4a^4b\left[a^2b(1-2b) + 4 + b\right] + a^2\left[2 + b(2+a)\right] \\
M_4 &= M_{10} = 2a^2\Big\{a\left[2 + b(2+a)\right](1+a^3b^3)+(1+2b)(1+2a^3b) \\
&\qquad + 4a^2b(2+b)\Big\} \\
M_5 &= M_9 = 2a^3\Big\{\left[2 + b(2+a)\right](2+a^2b) + 2(1+2b)\big[1 \\
&\qquad + a^2b(2+b)\big] + 2a^4b^3(2+b)\Big\} + 4a \\
M_6 &= M_8 = 2a^4\left[2 + b(2+a)\right]\left[1 + 2b + ab(2+b)\right] + 4a^3\big[2 \\
&\qquad + 4b + a^3b^2(2+b) + a^4b^3\big] + 8a^2 + 2a \\
M_7 &= 1 + 20a^2 + 4a^6b^2\left[a^2b^2 + 1 + (2+b)^2\right] + a^4\Big\{\big[2 \\
&\qquad + b(2+a)\big]^2 + 4(1+2b)^2\Big\} \\
N_o &= N_{14} = 2a^4b^2 \\
N_1 &= N_{13} = 4a^3b(1+a^2b) \\
N_2 &= N_{12} = 2a^2\Big\{1 + ab\left[2 + b + 4a(1+ab)\right]\Big\} \\
N_3 &= N_{11} = 2a^2\left[2 + b(2+a)\right] + 4a^3\Big\{1 + ab\big[6 + b \\
&\qquad + ab(1+a)\big]\Big\}
\end{aligned} \tag{17}$$

$$N_4 = N_{10} = (1+2a^3b)\left[2a(1+a) + 2a^2(1+2b)\right] + 8a^3\left[1 + ab(2+b)\right] + \left[2 + b(2+a)\right]\left[2a^3(2+a^3b^2)\right]$$

$$N_5 = N_9 = 8a + 4a^2(1+a)\left[1 + a^2b(2+b)\right] + 4a^3\left\{(1+2b)(1+a) + \left[2 + b(2+a)\right](2+a^2b) + a^3b^2\right\}$$

$$N_6 = N_8 = 4a + 16a^2 + 2a^3\left[2 + b(2+a)\right]\left[1 + a + a^2b(2+b) + 2a(1+b)\right] + 8a^2\left[1 + a + a(1+2b)\right] + 4a^5b\left[1 + a^2b^2 + ab(2+b)\right] \tag{17}$$

$$N_7 = 2 + 40a^2 + 8a^3\left[(1+a)(1+2b) + a^2b(ab + 2 + b)\right] + 2a^4\left[2 + b(2+a)\right]$$

of the ADPI coefficients in algebraic form was quite difficult. It required the product of two seventh order polynomials with algebraic coefficients. Because of this tedious task a computer program was written to eliminate the cataloging and tabulating errors that developed when the multiplying was done by hand. The program is illustrated in Appendix J. When the aidentity algorithm is applied to this ADPI, $M_o=N_o$ gives no information as they are identical and $M_1=N_1$ implies that O=1 for all values of m so that equation (16) will be a first order ADPI for all realizable values of m. Fritzemeyer (1) assumed m=0.6 and obtained the interactance response of Fig. 41. From the previous sections it seems that the m=0.707 would produce a better interactance response, but the ADPI being limited to a first order aidentity gives very little promise of much improvement. The aidentity algorithm could be used at this point, but Fritzemeyer (1)

suggested an additional process for complementary filter characteristic response improvement which will be called impedance elision as detailed in Appendix A. Impedance elision will yield the degenerate complementary configuration.

Type 6 Degenerate Complementary Zobel Filter with Parameters C and m

Applying impedance elision to the Type 5 Filter and adding the parameter C with m results in the complementary configuration of Fig. 42. The two parameters allow the aidentity algorithm to be used on the ADPI of equation (18). All values of O_o equal P_o because both are identical expressions. When O_1 is equated with P_1, $(1+m)C = (1+\sqrt{5})/2$ results and when O_2 is equated with P_2, $(1+m)C = 1/(1-m^2)$ results. These two simultaneous equations result in m=0.618033988 (Golden Mean) and C=1 and a third order ADPI. The numerical coefficients of equations (20) result when the determined values of m and C are substituted into the algebraic coefficients of equations (19). It should be noted that impedance

$$Z_{cf} = \frac{\sum_{n=o}^{10} O_n s^n}{\sum_{n=o}^{10} P_n s^n} \qquad (18)$$

where

$$a = 1 + m$$

$$b = 1 - m$$

$$O_o = O_{10} = 2Ca^3b$$

$$O_1 = O_9 = 2Ca^2\left[1 + a^2b(1+C)\right]$$

$$O_2 = O_8 = a^2\left\{b + C(2+b) + 2Ca\left[1 + C + ab(1+C+Ca)\right]\right\}$$

$$O_3 = O_7 = a(1+2Ca^2)(1+C+Ca) + 2Ca^5b(b+Cb+2C) + a^3(1+C)(b+2C+Cb)$$

$$O_4 = O_6 = 4C^2a^5b + 2Ca^4(b+Cb+2C) + a^2(1+C+Ca)\left[ab + aC(2+b) + 1 + C\right] + a + aC$$

$$O_5 = 4C^2a^4(1+a^2b^2) + a^4(b+bC+2C)(b+2C+Cb) + a^2(1+C+Ca)^2 + a^2(1+C)^2 + 1$$

$$P_o = P_{10} = 2Ca^3b \tag{19}$$

$$P_1 = P_9 = 2a^2\left\{b + C\left[1 + ab(1+a)\right]\right\}$$

$$P_2 = P_8 = 2a\left[1 + a^2b(1+C)\right] + 2Ca^2\left[1 + a + a^2b(2+b)\right] + a^2(b+2C+Cb)$$

$$P_3 = P_7 = (1+C+Ca)(a+2a^3b) + 2a^2(1+C) + 2a + ab + 2Ca^3(2+b+2ab) + a^2(1+a)(b+2C+Cb)$$

$$P_4 = P_6 = 2a^4b(b+Cb+2C) + (1+C+Ca)(a+3a^2) + a^2(2+b)\left[1 + C + a(b+2C+Cb)\right] + 4Ca^3(1+a^2b^2) + 2a + Ca + 1$$

$$P_5 = 2\left[1 + a(1+C)(1+a) + a^2(2+b)(1+C+Ca) + 4Ca^4b + 2a^3(b+Cb+2C)\right]$$

$O_o = O_{10} = 3.236$	$P_o = P_{10} = 3.236$
$O_1 = O_9 = 15.708$	$P_1 = P_4 = 15.708$
$O_2 = O_8 = 43.125$	$P_2 = P_8 = 43.125$ (20)
$O_3 = O_7 = 83.339$	$P_3 = P_7 = 81.485$
$O_4 = O_6 = 119.374$	$P_4 = P_6 = 118.374$
$O_5 = 135.992$	$P_5 = 131.846$

elision and added C parameter have increased the aidentity order by two. The frequency response characteristics are given in Fig. 43 and Fig. 44 for this degenerate complementary configuration. The interactance has been greatly improved by impedance elision and the aidentity algorithm as compared with Type 5. This filter has by far the best low-pass VTF response of any type presented. At w_c it has a value of 0.32 and reaches 0.1 at w=1.08 rad/sec. The low-pass VTF phase shift is nearly linear to approximately 0.8 rad/sec but then displays the usual nonlinearities about w_c. The ADPI phase shift is nearly constant from zero degrees until 0.8 rad/sec but then becomes nonlinear about w_c as has been the character in the previous types. These low-pass characteristics substantiate the thinking behind the use of impedance elision as they are very nearly the same as those of Fig. 16 and Fig. 17 for the Zobel filter of Fig. 15. Also the high-pass series arm impedance nearest the input terminals of Type 6 is 1.618/s where the shunt capacitive impedance of Fig. 15 is 1.414/s. And the high-pass shunt arm impedance nearest the input terminals of Type 6 is s/2

where the shunt inductive impedance of Fig. 15 is s/1.414. This implies that the first series impedance and shunt impedance of the high-pass complementary filter of the degenerate Zobel filter takes the place of the deleted branch in the low-pass complementary filter. The characteristic responses of this elision or sluring have been shown to be quite favorable and that it is these first two impedances that appear to have the most effect on the other complementary filter.

Type 7 Degenerate Complementary Zobel Filter with Parameters C, C_1 and m

An additional attempt was made to improve the ADPI of the Type 6 degenerate complementary configuration by the addition of another circuit parameter, C_1, as is shown in Fig. 45. From this network the ADPI of equation (21) is obtained with the algebraic coefficients of equations (22). Obtaining these coefficients required a considerable amount of toil similar to that experienced with Type 6. Applying the aidentity algorithm to these coefficients results in $Q_o = R_o$ which gives no information as both expressions are identical. Since there are three parameters, at least three other sets of equations (22) coefficients will have to be equated. The result of Q_1 equated with R_1 is equation (23), of Q_2 equated with R_2 is equation (24), and of Q_3 equated with R_3 is equation (25).

$$Z_{cf} = \frac{\sum_{n=o}^{10} Q_n s^n}{\sum_{n=o}^{10} R_n s^n} \tag{21}$$

where

$a = 1 + m$

$b = 1 - m$

$$\begin{aligned}
Q_o &= Q_{10} = a^3 bCC_1 \\
Q_1 &= Q_9 = a^4 bCC_1(1+C) + a^2 CC_1 \\
Q_2 &= Q_8 = a^4 bC\left[C + C_1(1+Cm)\right] + a^3 CC_1(1+C) + a^2 C_1\left[C + bC_1(1+C)\right] \\
Q_3 &= Q_7 = a^5 bC\left[C + bC_1(1+C)\right] + a^3 C\left[C + C_1(1+Cm)\right] \\
&\quad + a^3 C_1(1+C)\left[C + bC_1(1+C)\right] + aC_1\left[C + C_1(1+Cm)\right] \\
Q_4 &= Q_6 = a^5 bC_1{}^2 + a^4 C\left[C + bC_1(1+C)\right] + a^3\left[C + bC_1(1+C)\right]\left[C + C_1(1+Cm)\right] + a^2 C_1(1+C)\left[C + C_1(1+Cm)\right] + aC_1{}^2(1+C) \\
Q_5 &= a^6 b^2 C^2 + a^4 C^2 + a^4\left[C + bC_1(1+C)\right]^2 + a^2\left[C + C_1(1+Cm)\right]^2 + a^2 C_1{}^2(1+C)^2 + C_1{}^2 \\
R_o &= R_{10} = a^3 bCC_1 \\
R_1 &= R_9 = a^3 bC(1+C_1 m) + a^2 CC_1 + a^2 bC_1 \\
R_2 &= R_8 = a^4 bC(1+bC_1) + a^2 C(1+C_1 m) + a^2 C_1\left[C + bC_1(1+C)\right] + a^3 bC_1(1+C) + aC_1
\end{aligned} \tag{22}$$

$$R_3 = R_7 = a^4bC + a^3C(1+bC_1) + a^2(1+C_1m)\left[C + bC_1(1+C)\right]$$
$$+ aC_1\left[C + C_1(1+Cm)\right] + a^3b\left[C + C_1(1+Cm)\right]$$
$$+ a^2C_1(1+C) + aC_1(1+bC_1)$$
$$R_4 = R_6 = a^5b^2C + a^3C + a^3(1+bC_1)\left[C + bC_1(1+C)\right] \tag{22}$$
$$+ a(1+C_1m)\left[C + C_1(1+Cm)\right] + aC_1^2(1+C)$$
$$+ a^4b\left[C + bC_1(1+C)\right] + a^2\left[C + C_1(1+Cm)\right]$$
$$+ a^2C_1(1+C)(1+bC_1) + C_1(1+C_1m)$$
$$R_5 = 2\left\{a^4bC + a^3\left[C + bC_1(1+C)\right] + a^2(1+bC_1)\left[C + C_1(1\right.\right.$$
$$\left.\left.+Cm)\right] + aC_1(1+C)(1+C_1m) + C_1^2\right\}$$

$$aC(C_1+aCC_1-1) - C_1 = 0 \tag{23}$$

$$a^3bC\left[C-1+C_1m(1+C)\right]+a^2C_1(1+C)(C-b)-aC(1+C_1m)-C_1 = 0 \tag{24}$$

$$a^4bC\left[C + bC_1(1+C)\right] + a^3C\left[CC_1 - b(1+C_1)\right] + a^2\left\{CC_1\left[2\right.\right.$$
$$\left.\left. + bC_1(2+C)\right] + b(C_1^2-C_1-C) - C + C^2\right\} + a\left\{C\right. \tag{25}$$
$$\left. + C_1\left[b(1+C_1m+CC_1m) - 1\right]\right\} - C_1(1+bC_1) = 0$$

The method of solving these three simultaneous equations is rather laborious so it has been placed in Appendix K. Their solutions give the following values for C, C_1, and m:

$$C = 0.92041$$
$$C_1 = 0.51437$$
$$m = 0.71433$$

With these quantities and the computer program of Appendix C,

the ADPI coefficients of equations (22) were evaluated and these results are given in equations (26). This shows that

$$
\begin{aligned}
Q_o &= Q_{10} = 0.6814 & \qquad R_o &= R_{10} = 0.6814 \\
Q_1 &= Q_9 = 3.6347 & R_1 &= R_9 = 3.6347 \\
Q_2 &= Q_8 = 10.4251 & R_2 &= R_8 = 10.4251 \\
Q_3 &= Q_7 = 20.4523 & R_3 &= R_7 = 20.4523 \\
Q_4 &= Q_6 = 29.9044 & R_4 &= R_6 = 29.9370 \\
Q_5 &= 33.9341 & R_5 &= 33.7731
\end{aligned}
\tag{26}
$$

the ADPI aidentity order has been increased by one over Type 6 to a fourth order. Examination of Q_4 and R_4 reveals that it is very nearly a fifth order aidentity. The filter frequency response characteristics are given in Fig. 46 and Fig. 47. The small improvement of the frequency response characteristics of this type over Type 6 is very similar to the small improvement that Type 3 had over Type 2. In both of these instances the added parameter was in the shunt arms. This would seem to indicate that the shunt arms do not have as much influence on the frequency response characteristics as do the series arms. As in Type 4 when the additional parameter was placed in the remaining series arm the characteristics improved quite significantly, it is believed that the same improvements would be observed in this filter. However it should be noted that solving for the four parameters would be a tremendous task beyond the scope of this thesis. The frequency response characteristics of this filter contradict Fritzemeyer's (1)

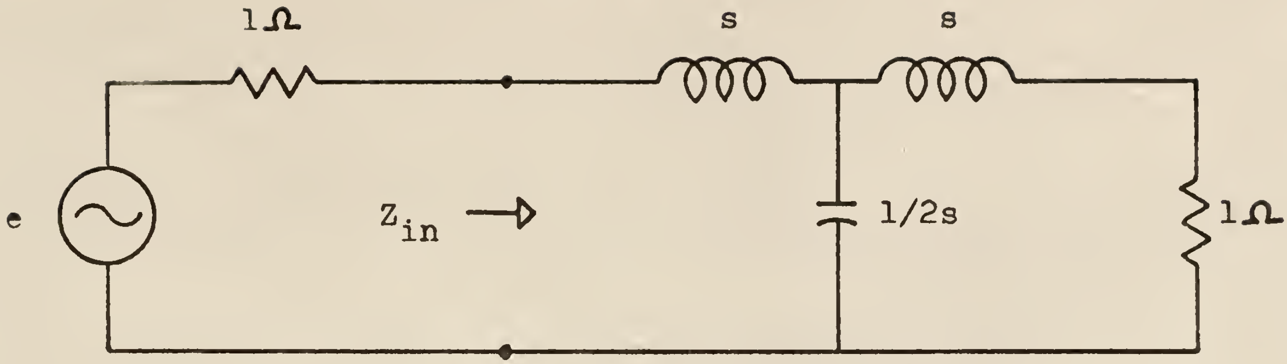


Fig. 19. Constant-l filter.

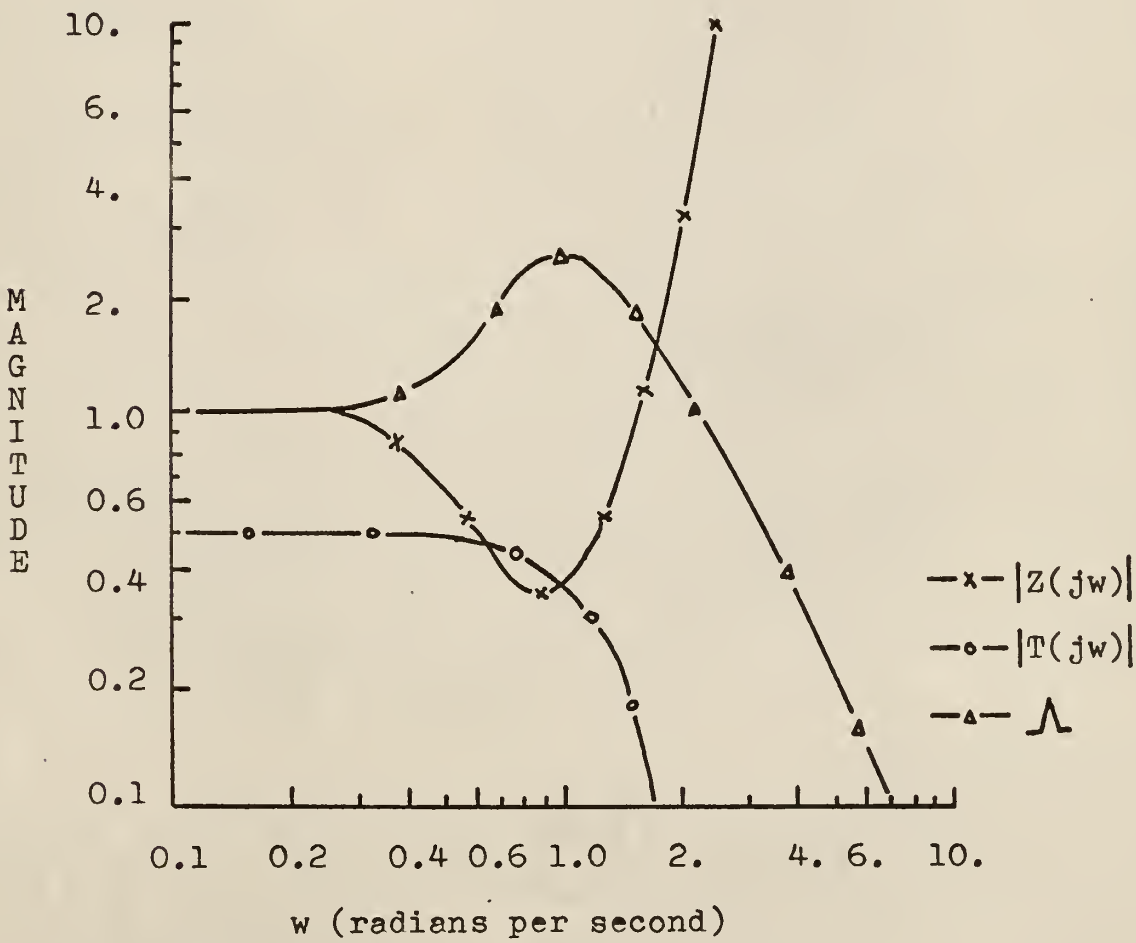


Fig. 20. Constant-l filter characteristics.

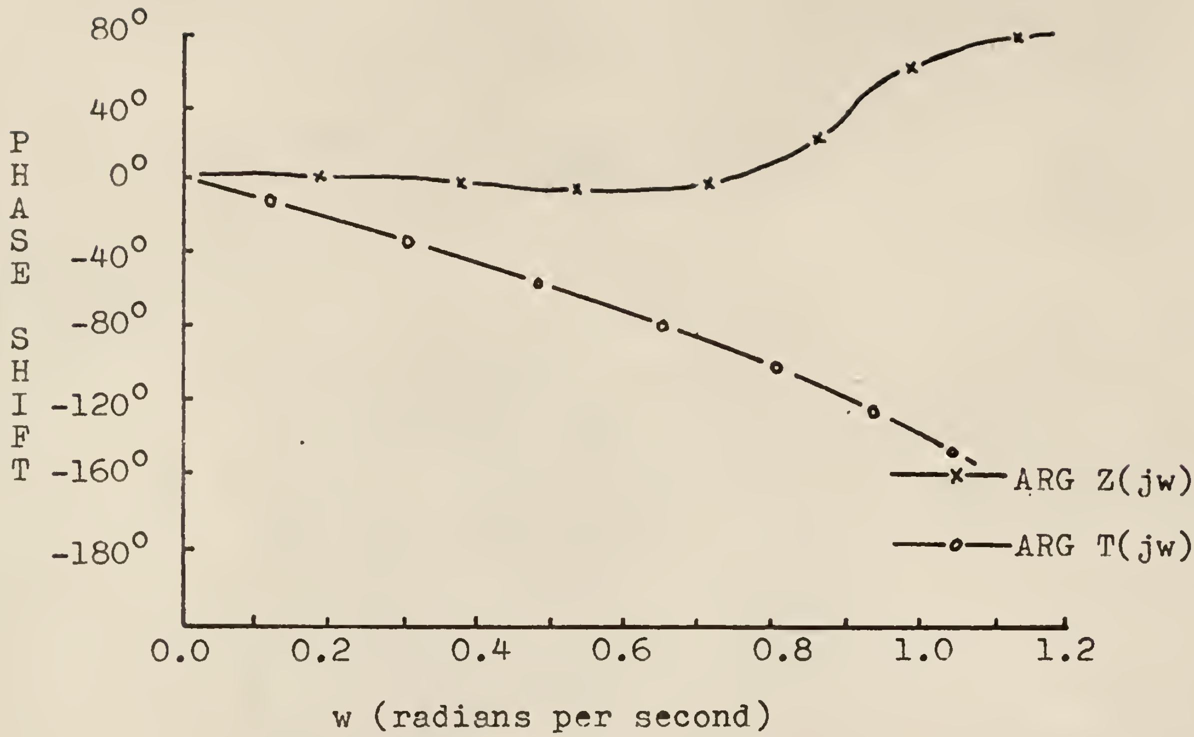


Fig. 21. Constant-1 filter characteristics.

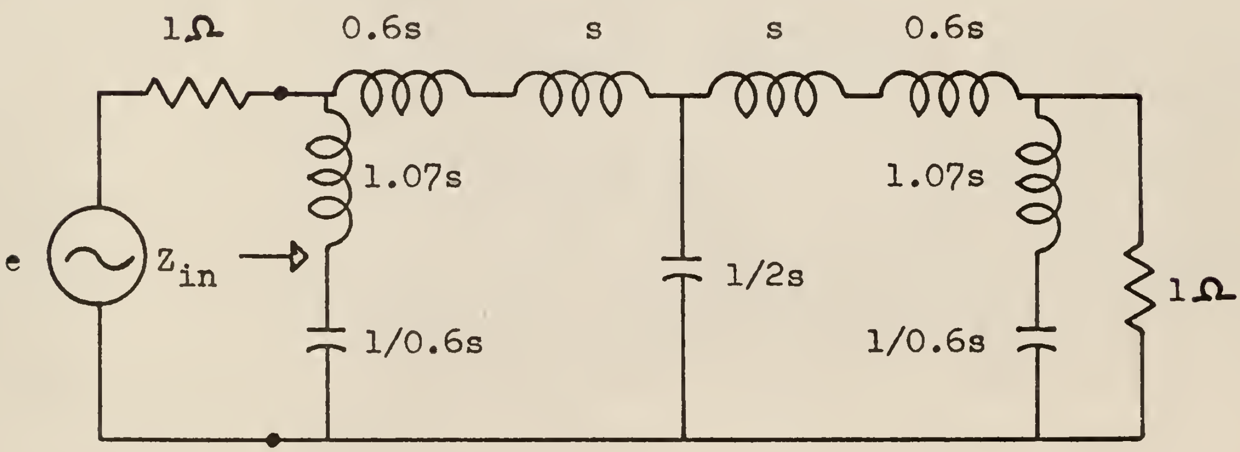


Fig. 22. Zobel filter with m=0.6.

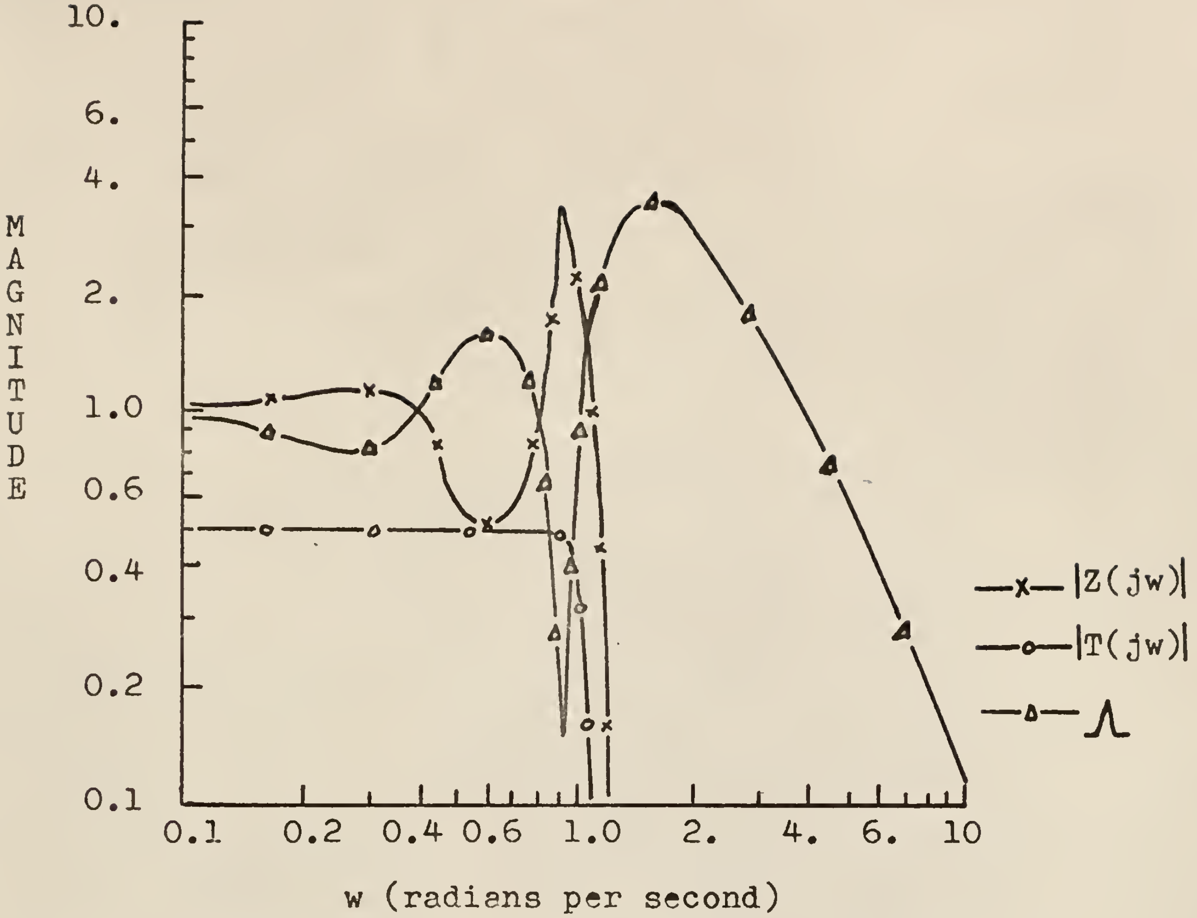


Fig. 23. Zobel filter characteristics with m=0.6.

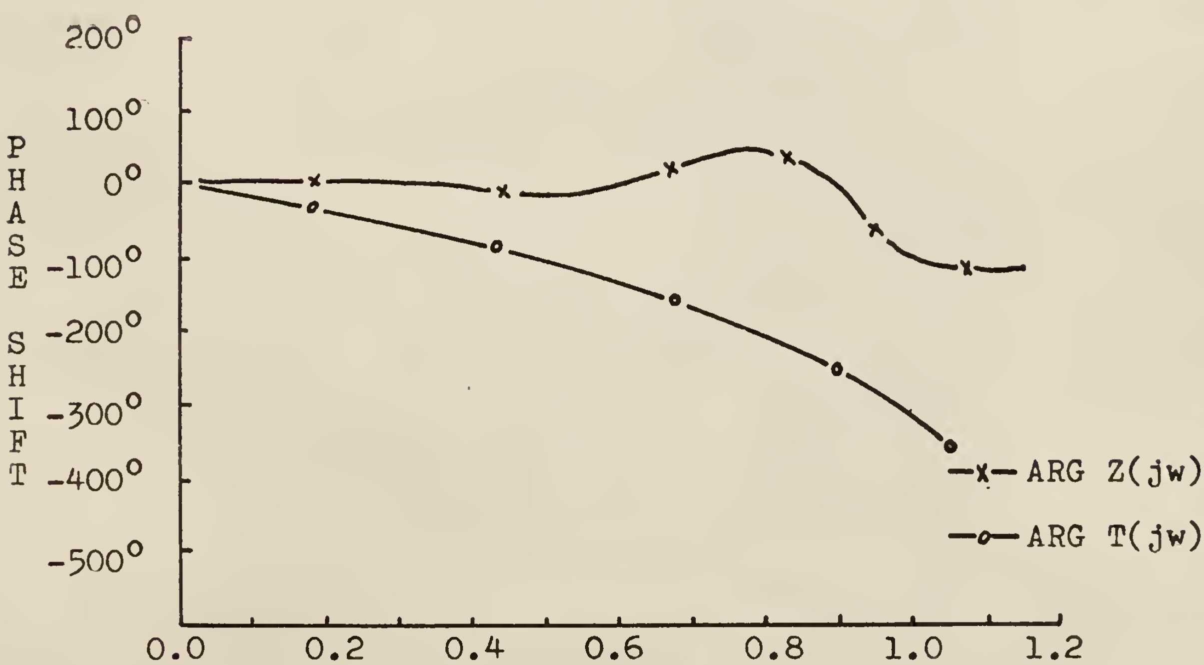


Fig. 24. Zobel filter characteristics with m=0.6.

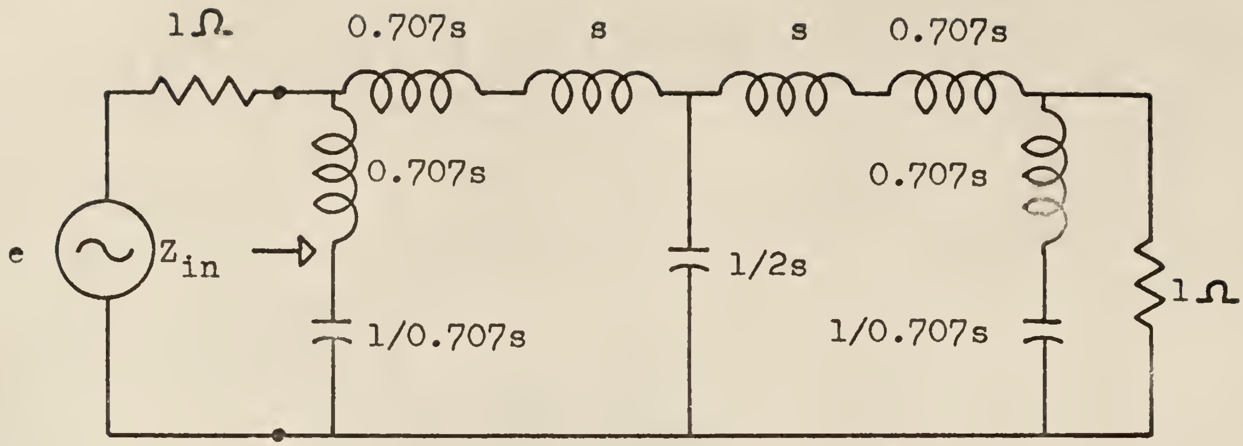


Fig. 25. Zobel filter with m=0.707.

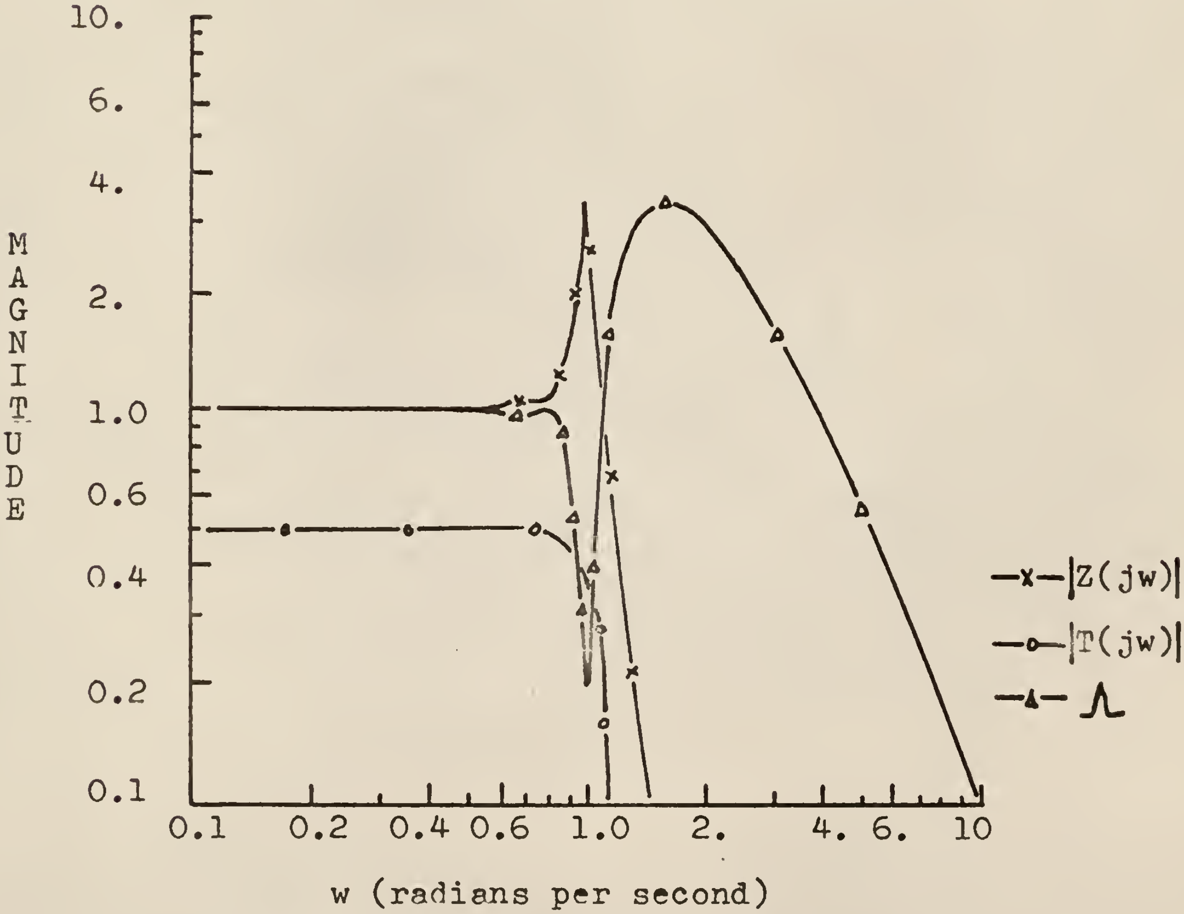


Fig. 26. Zobel filter characteristics with m=0.707.

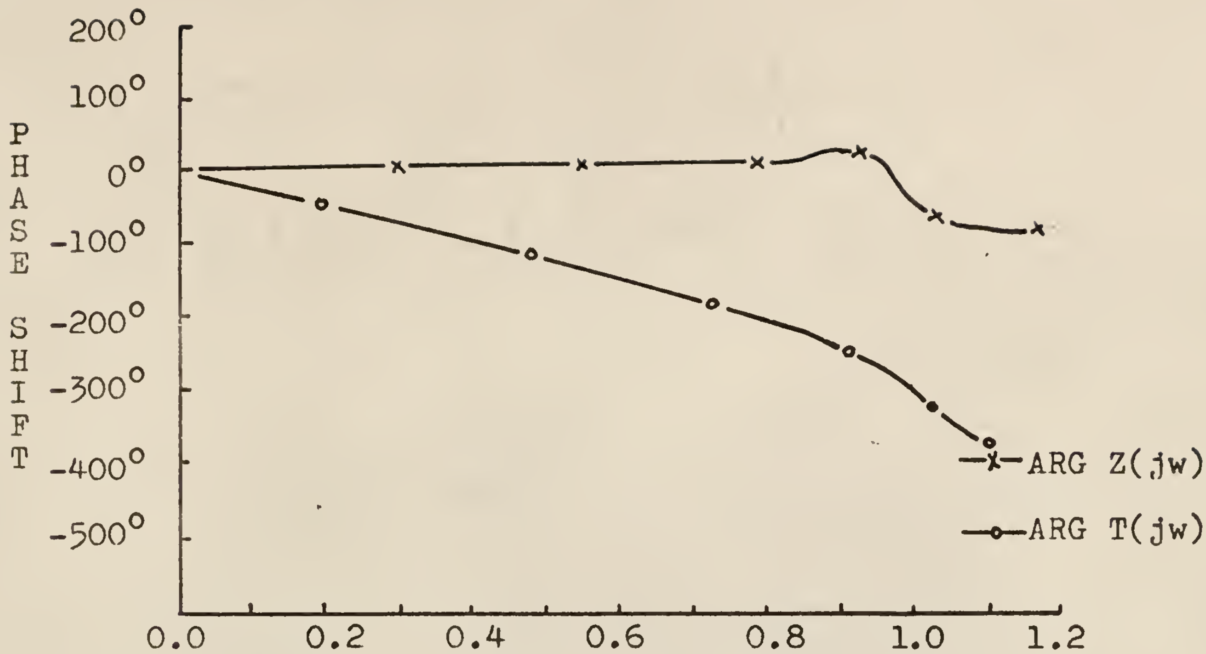


Fig. 27. Zobel filter characteristics with m=0.707.

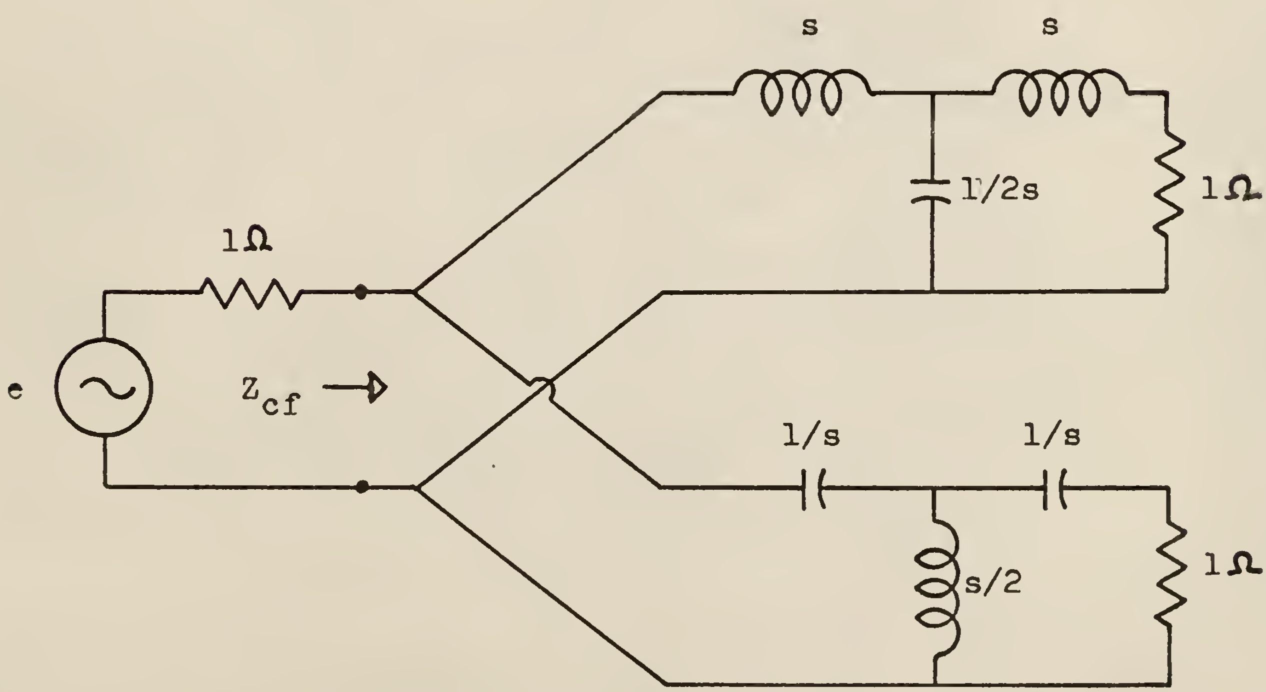


Fig. 28. Type 1 constant-1 complementary filter.

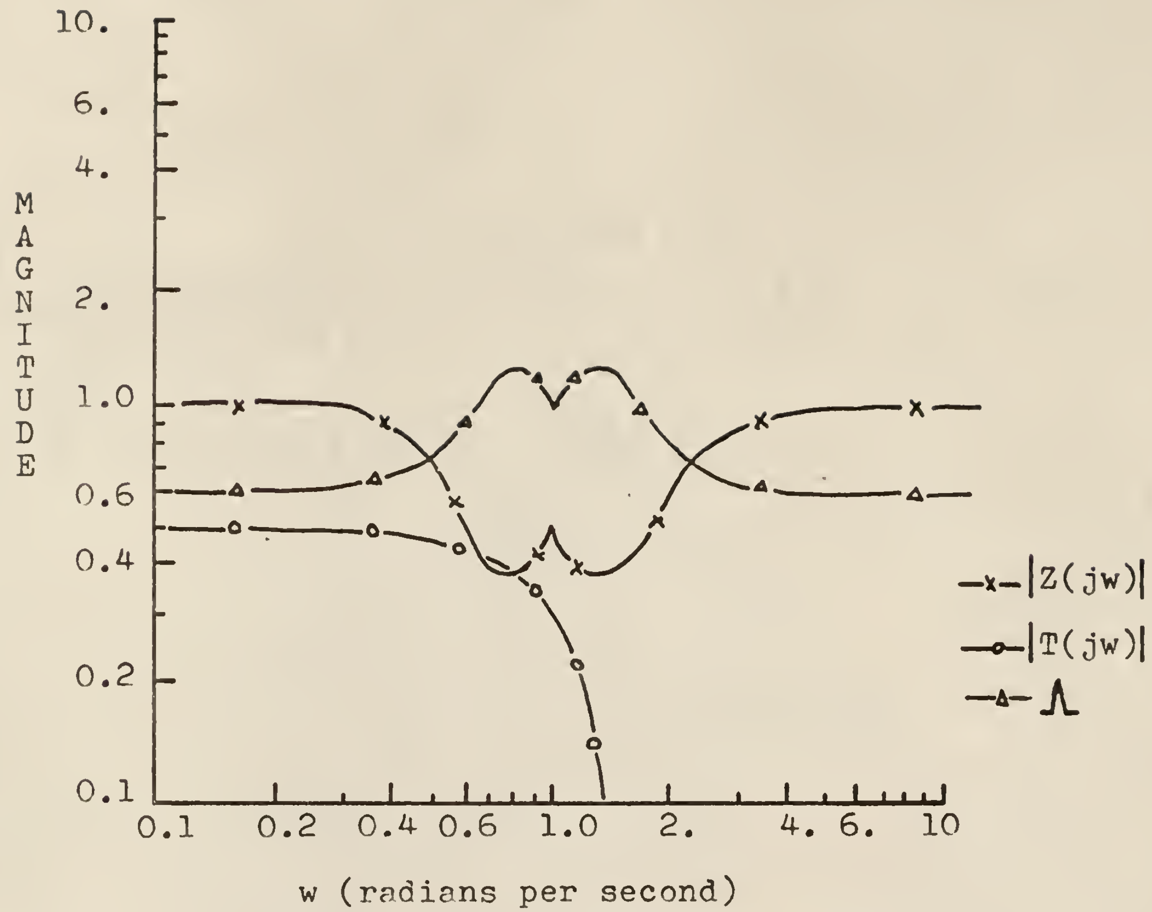


Fig. 29. Type 1 frequency response characteristics.

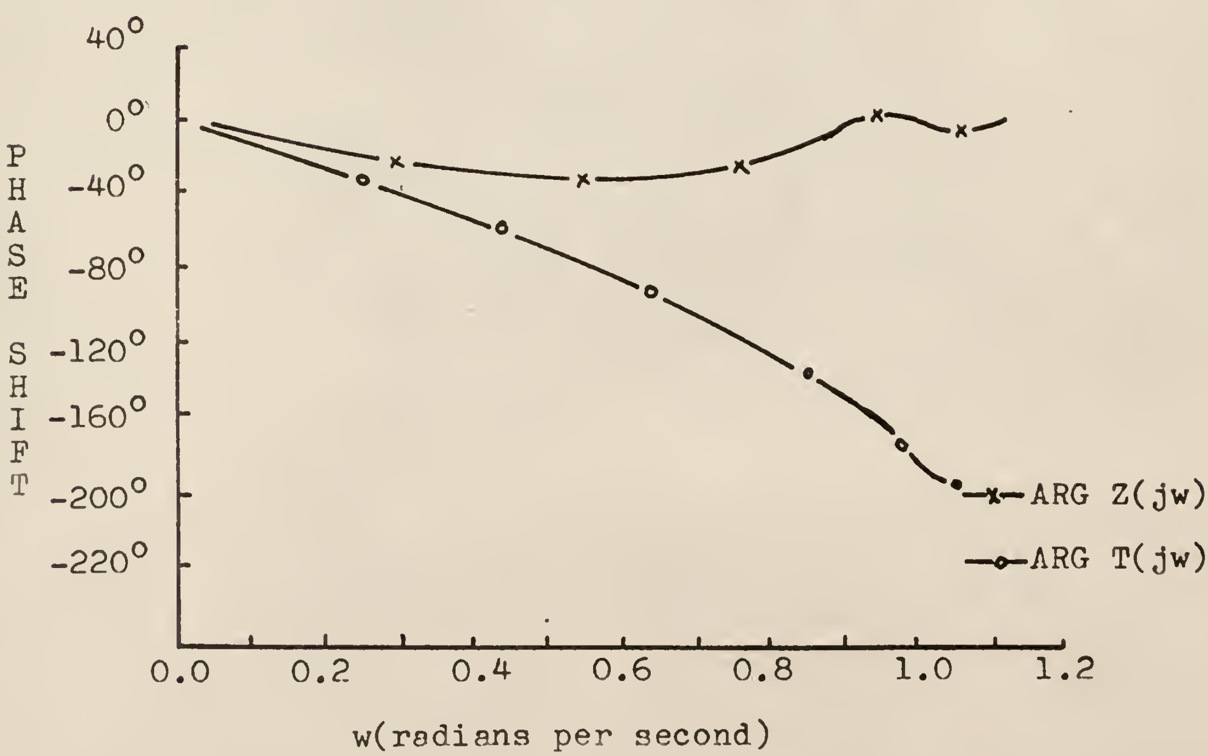


Fig. 30. Type 1 frequency response characteristics.

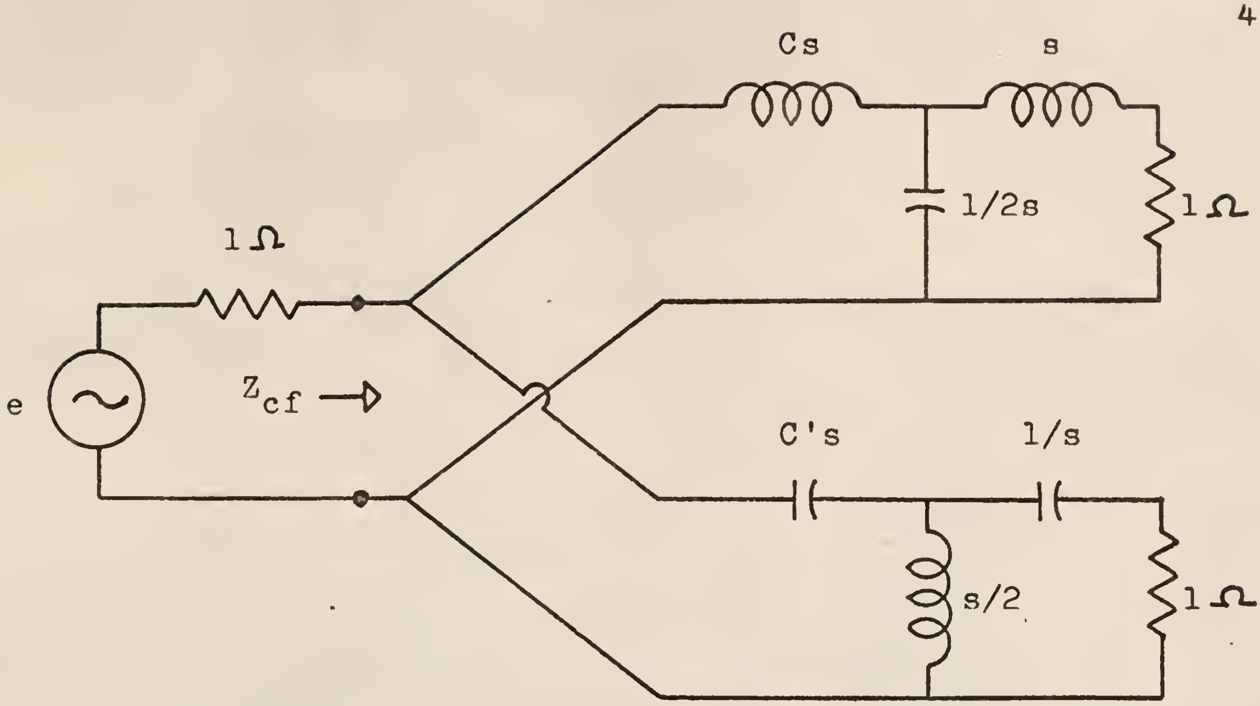


Fig. 31. Type 2 constant-l complementary filter with parameter C and C'.

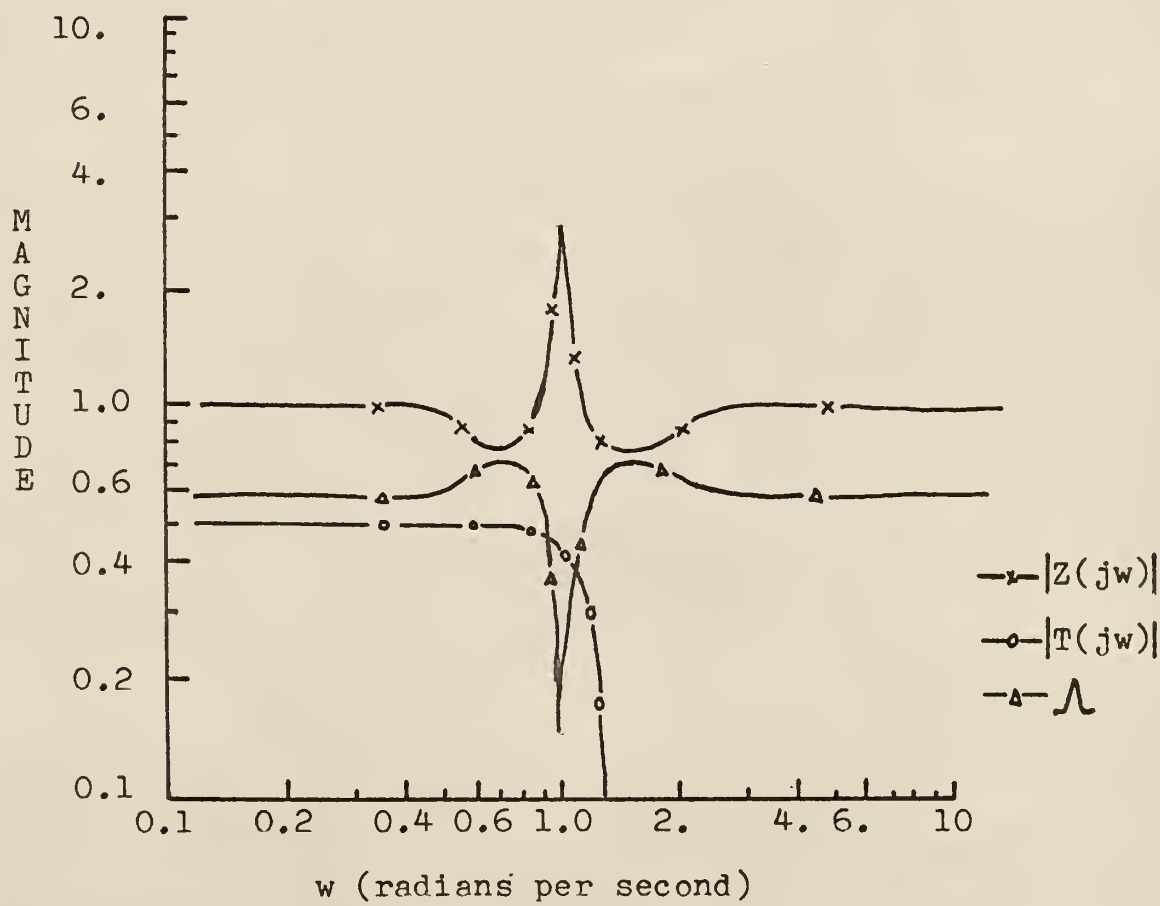


Fig. 32. Type 2 frequency response characteristics.

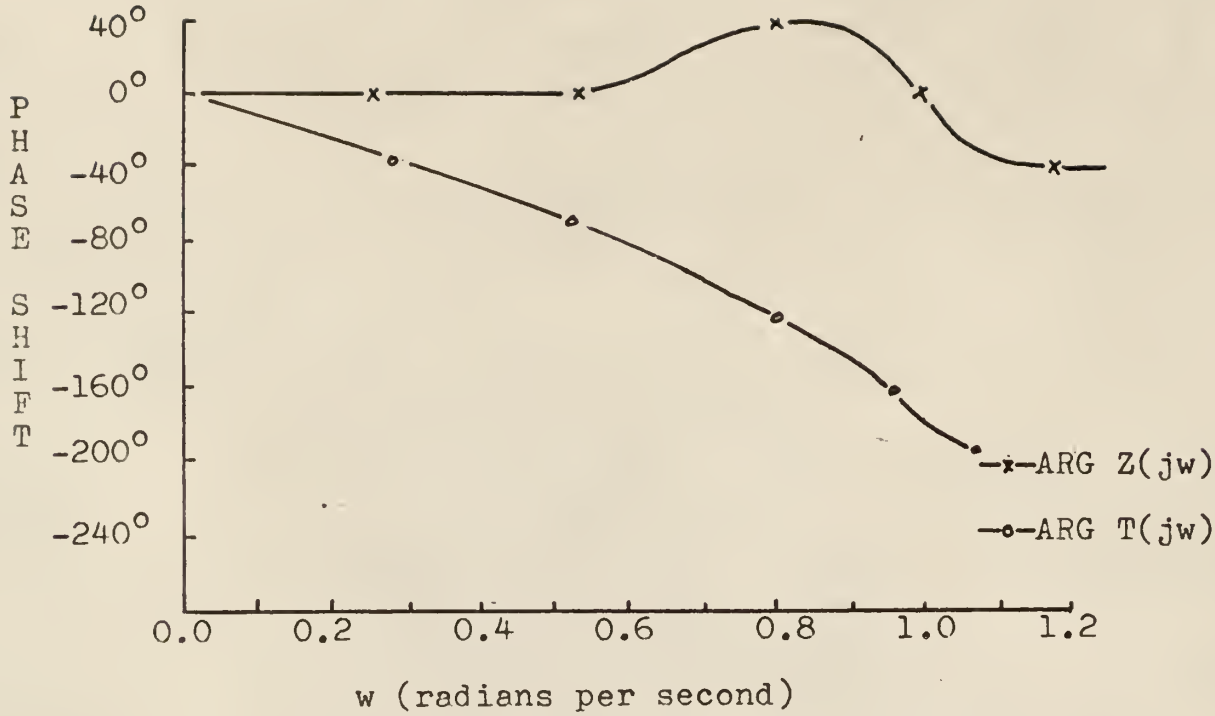


Fig. 33. Type 2 frequency response characteristics.

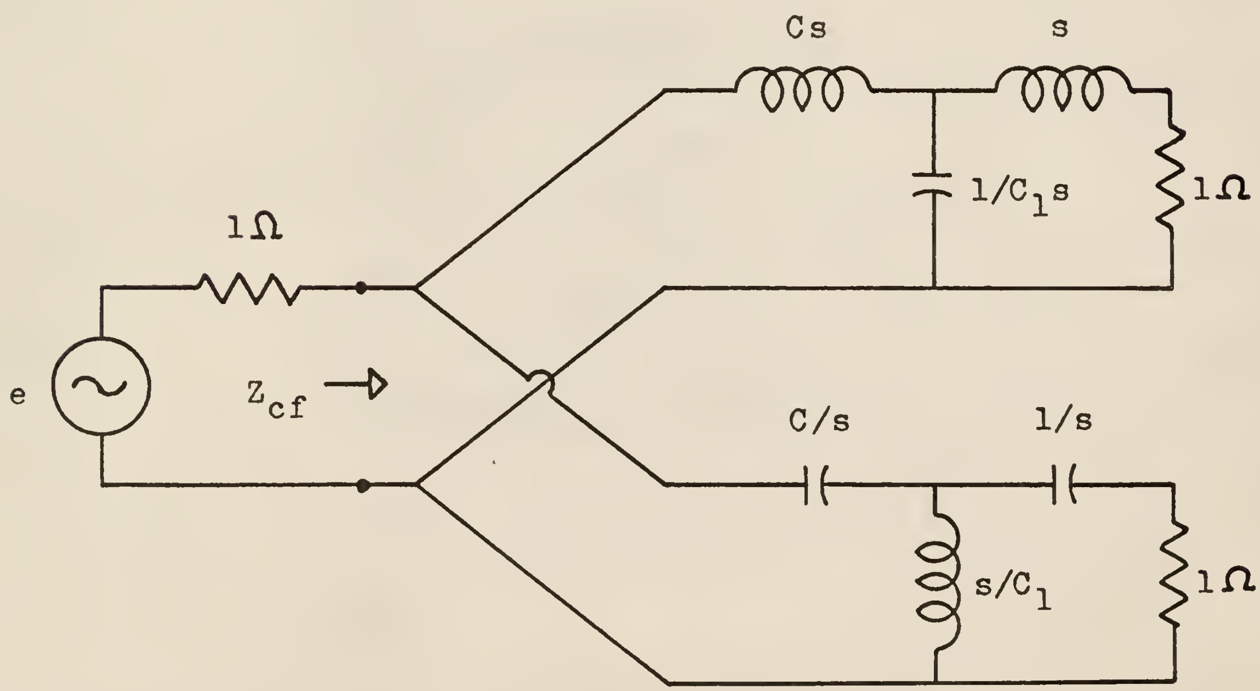


Fig. 34. Type 3 constant-1 complementary filter with parameters C and C_1.

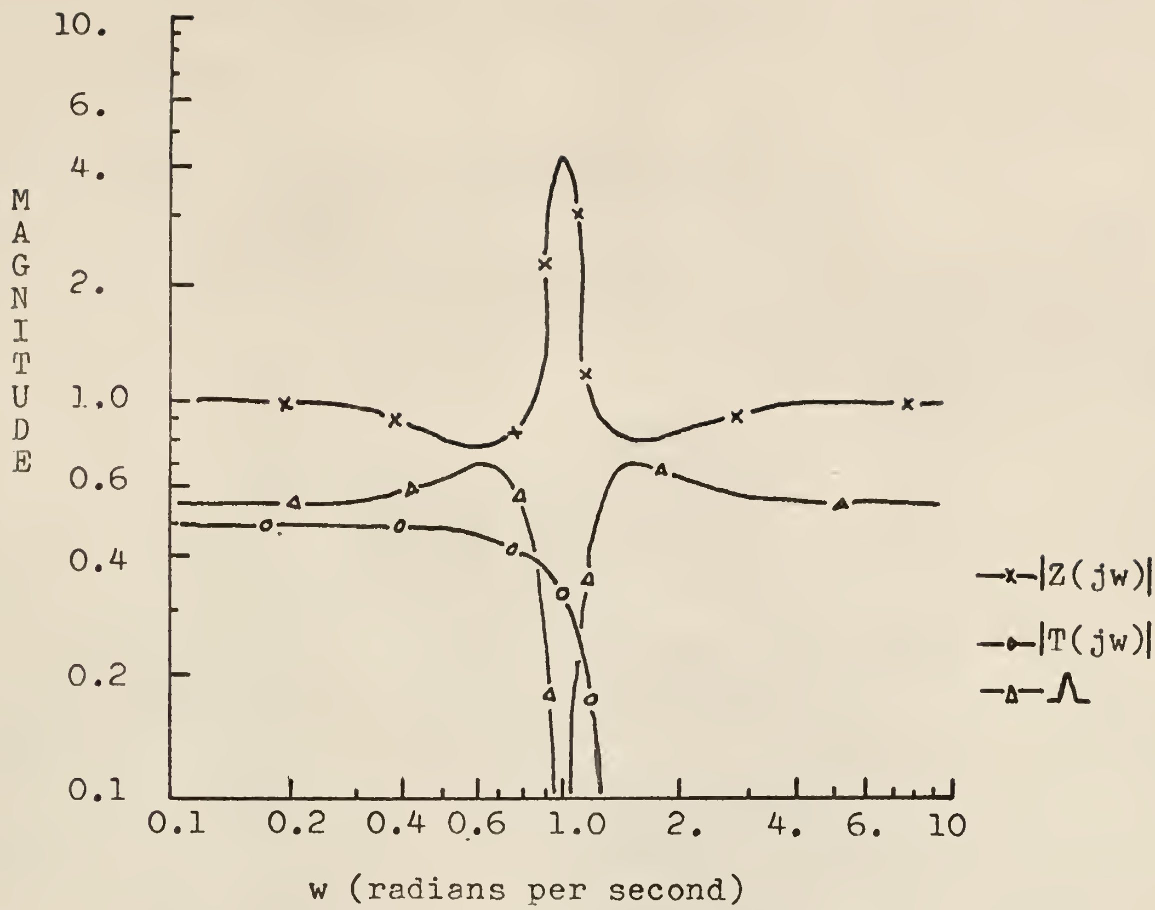


Fig. 35. Type 3 frequency response characteristics.

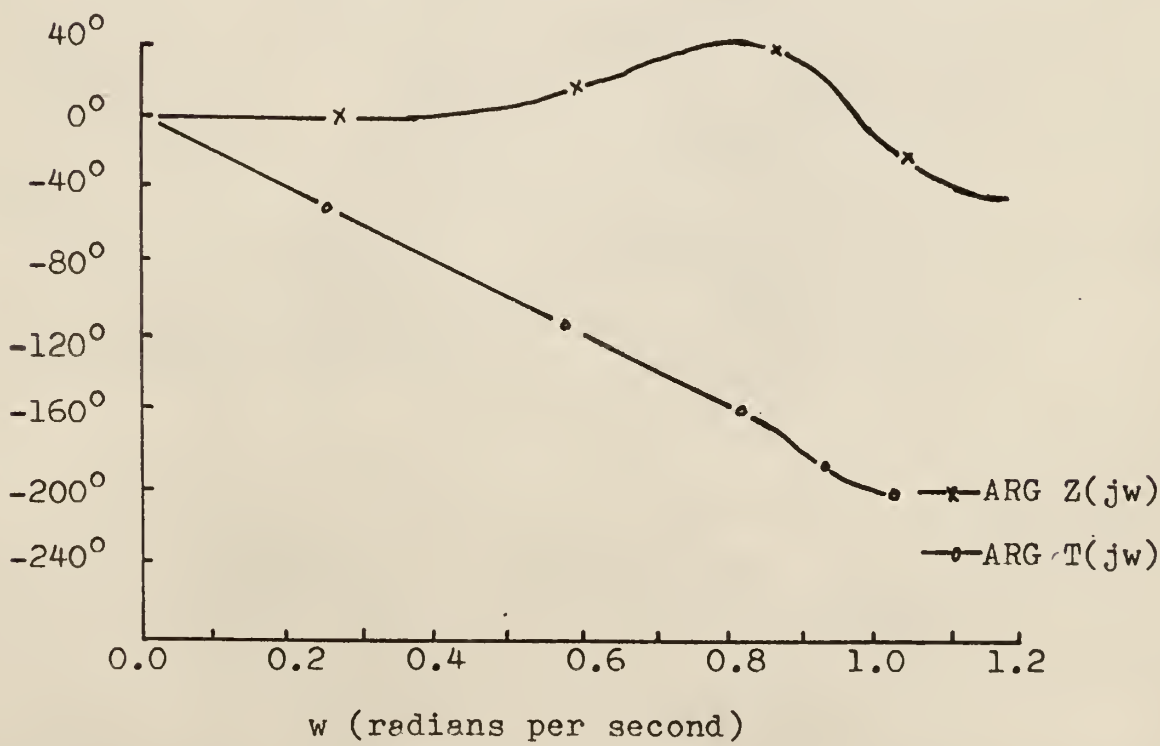


Fig. 36. Type 3 frequency response characteristics.

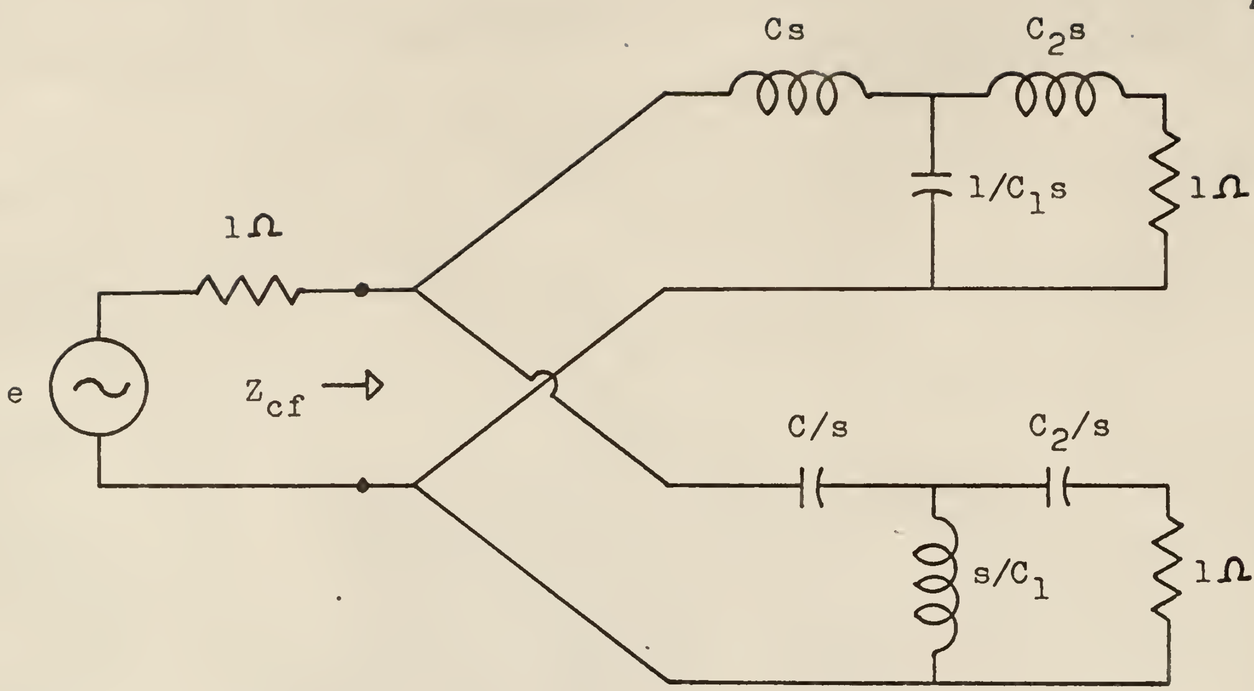


Fig. 37. Type 4 constant-1 complementary filter with parameters C, C_1 and C_2.

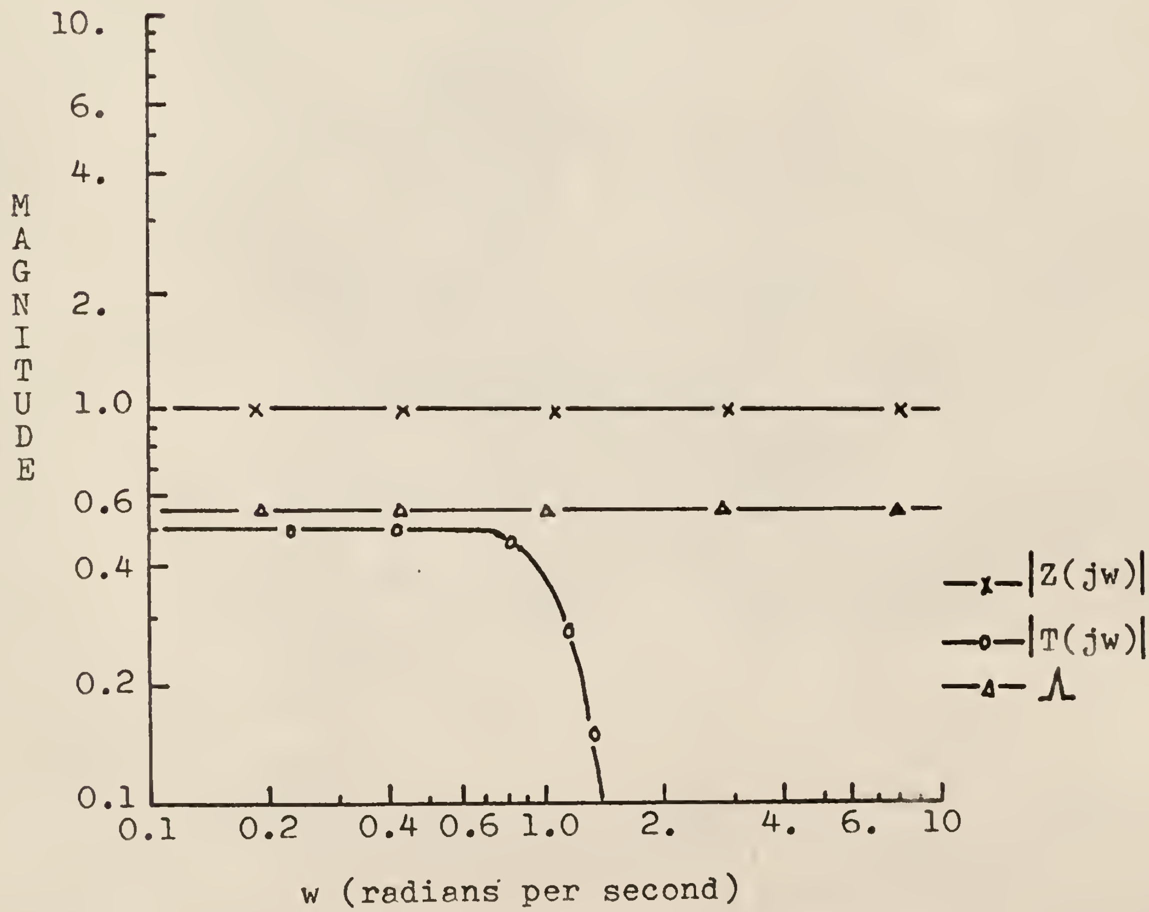


Fig. 38. Type 4 frequency response characteristics.

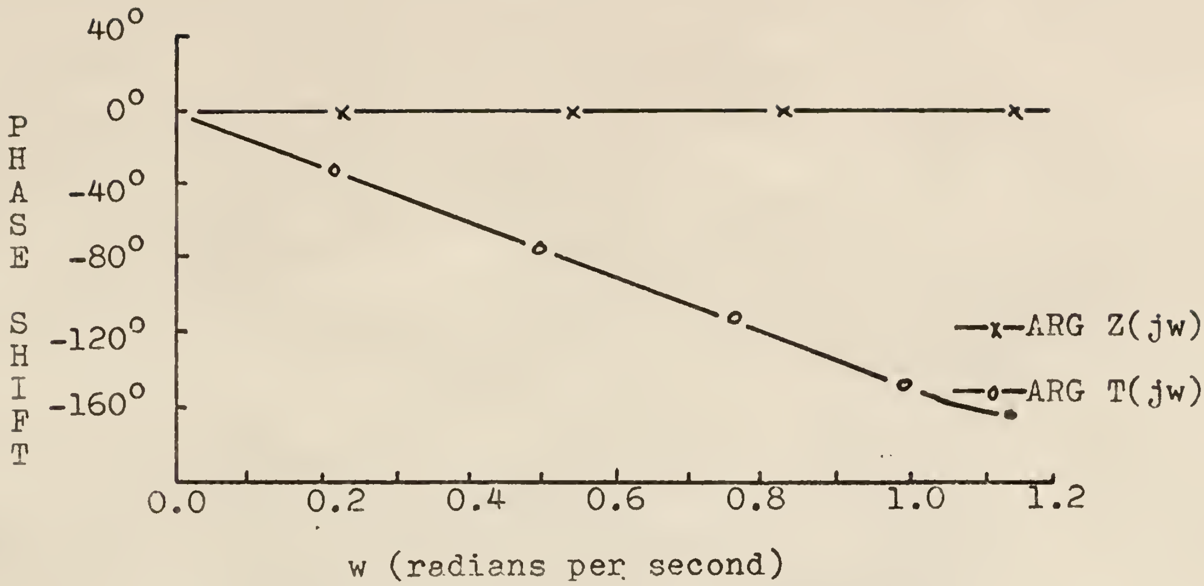


Fig. 39. Type 4 frequency response characteristics.

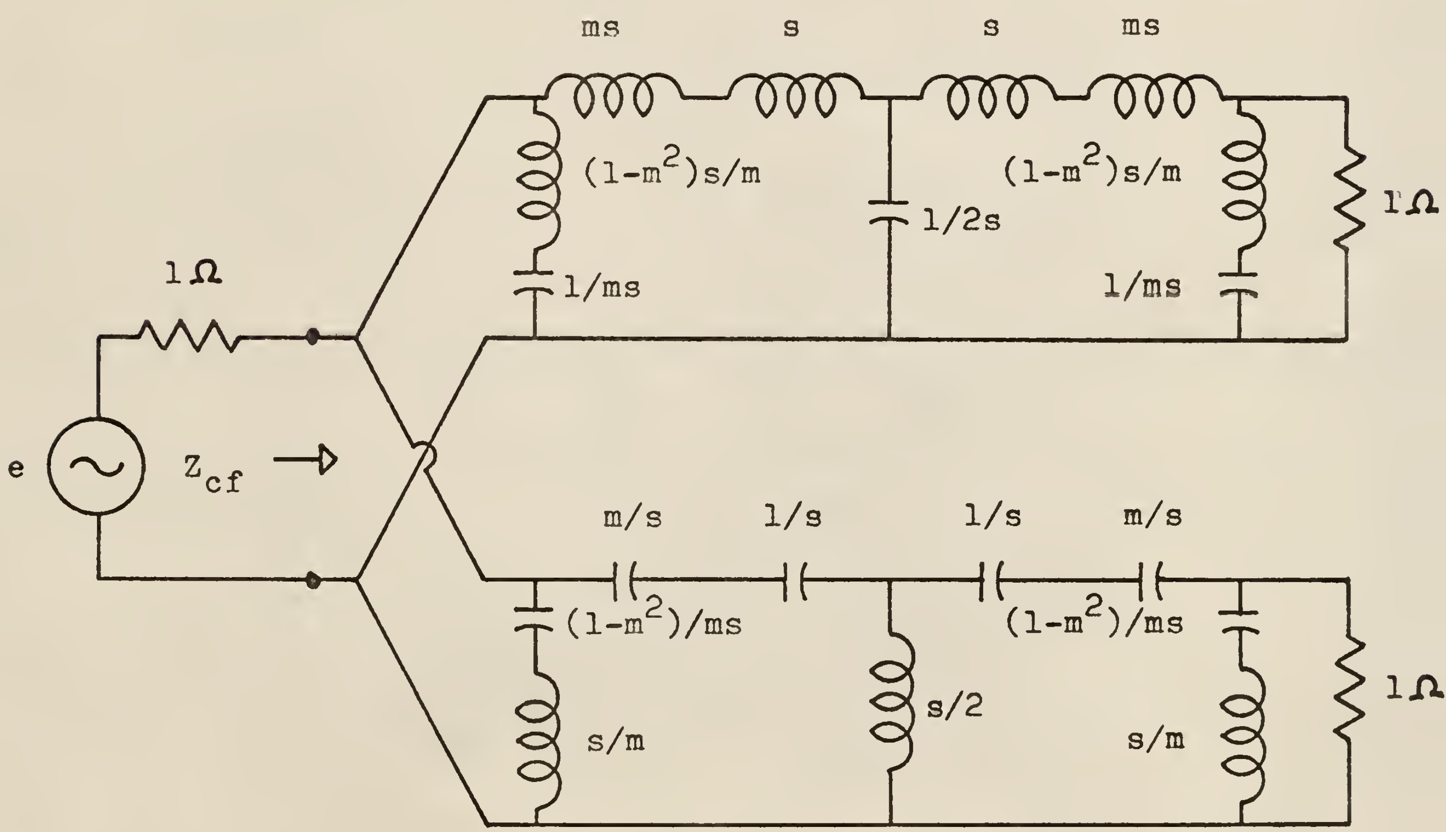


Fig. 40. Type 5 Complimentary Zobel filter.

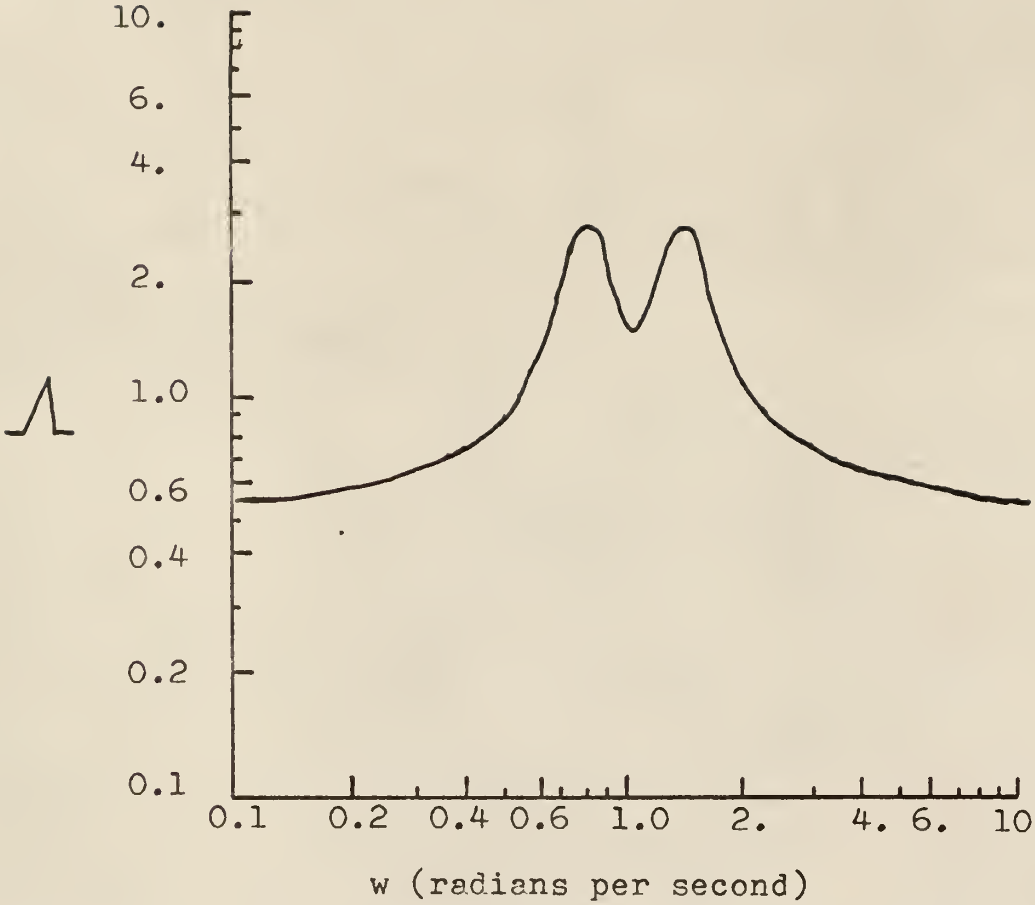


Fig. 41. Interactance of Type 5 complementary Zobel filter with m=0.6.

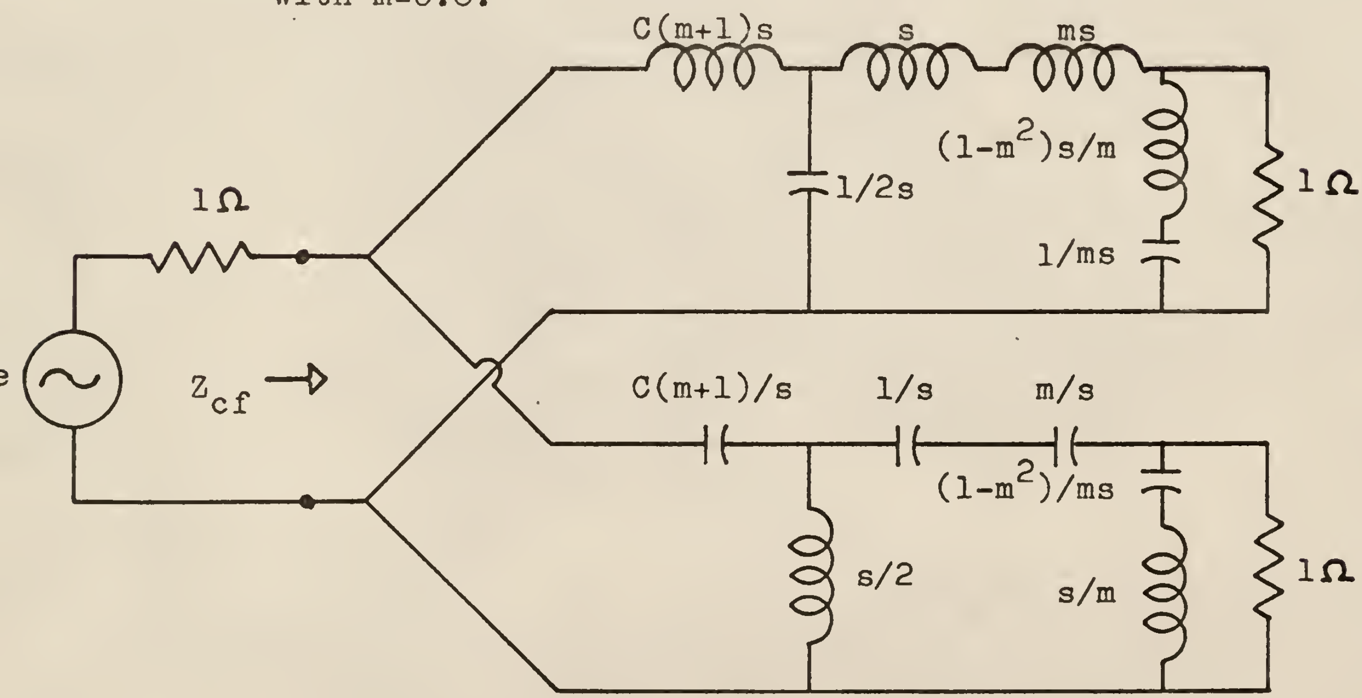


Fig. 42. Type 6 degenerate complementary Zobel filter with parameters C and m.

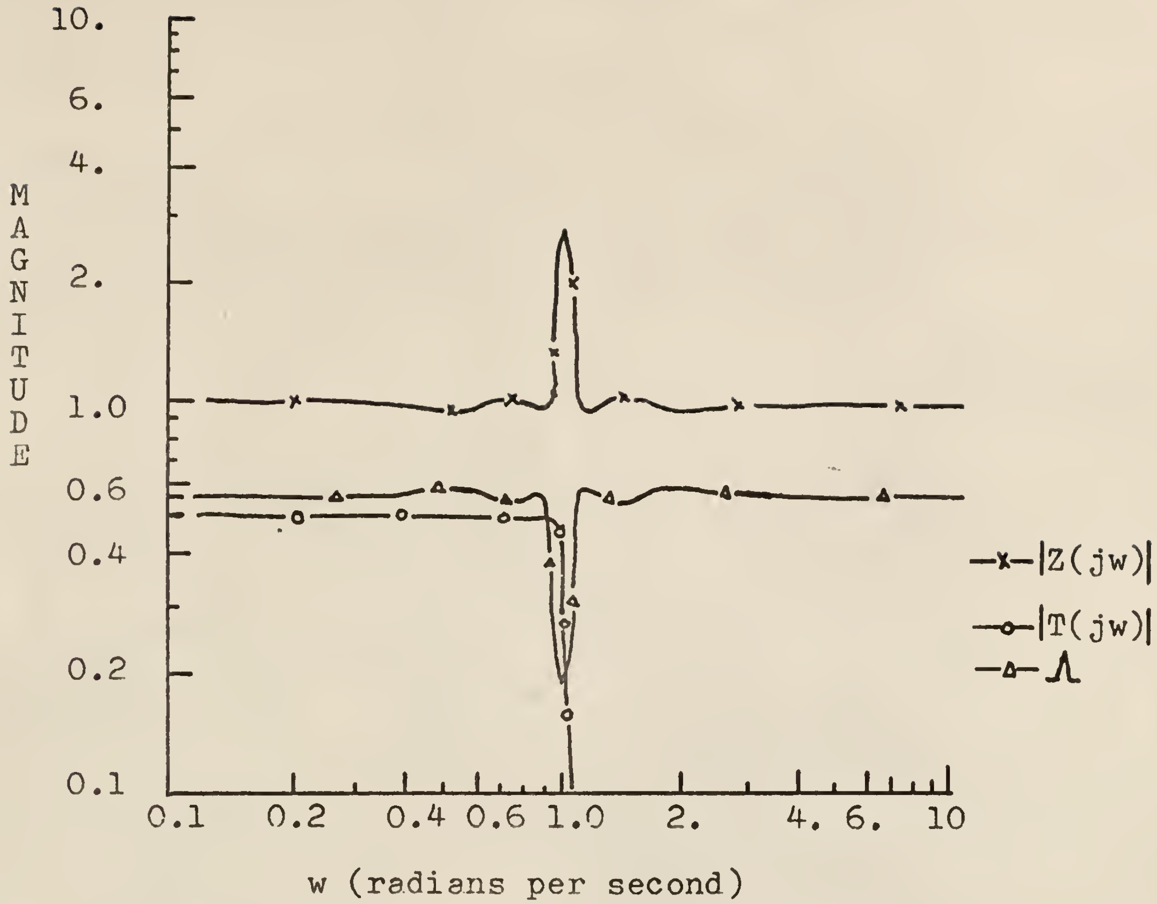


Fig. 43. Type 6 frequency response characteristics.

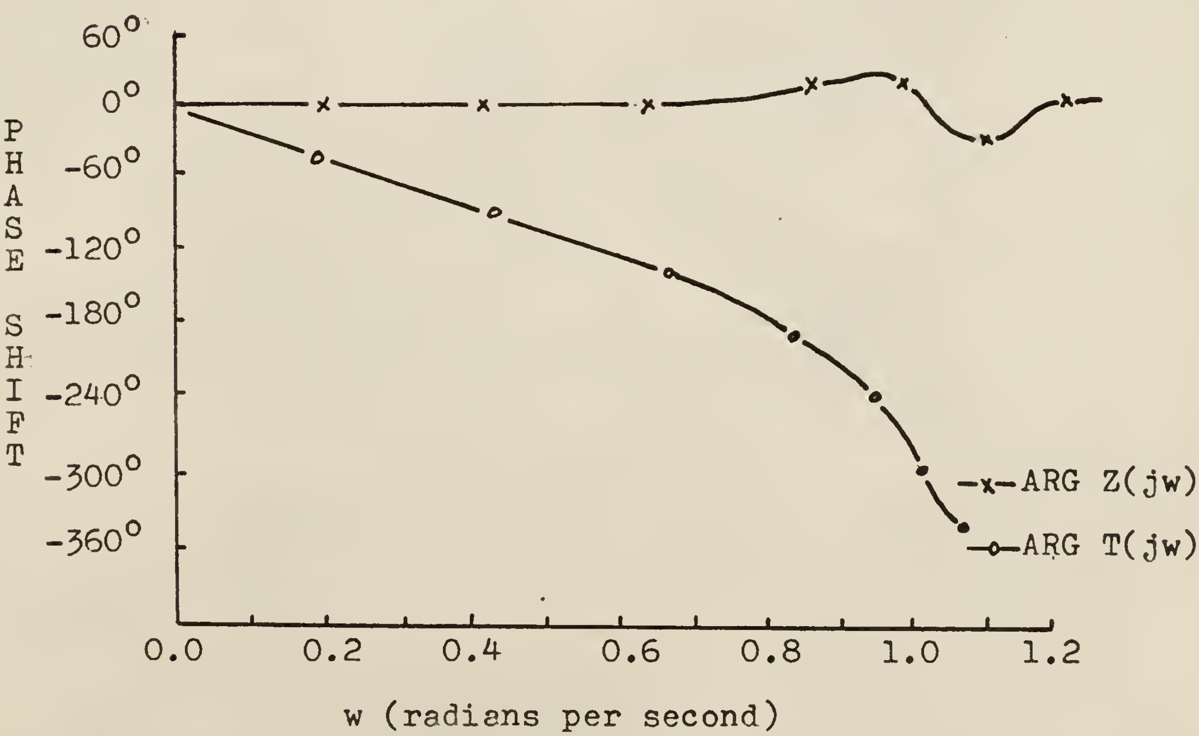


Fig. 44. Type 6 frequency response characteristics.

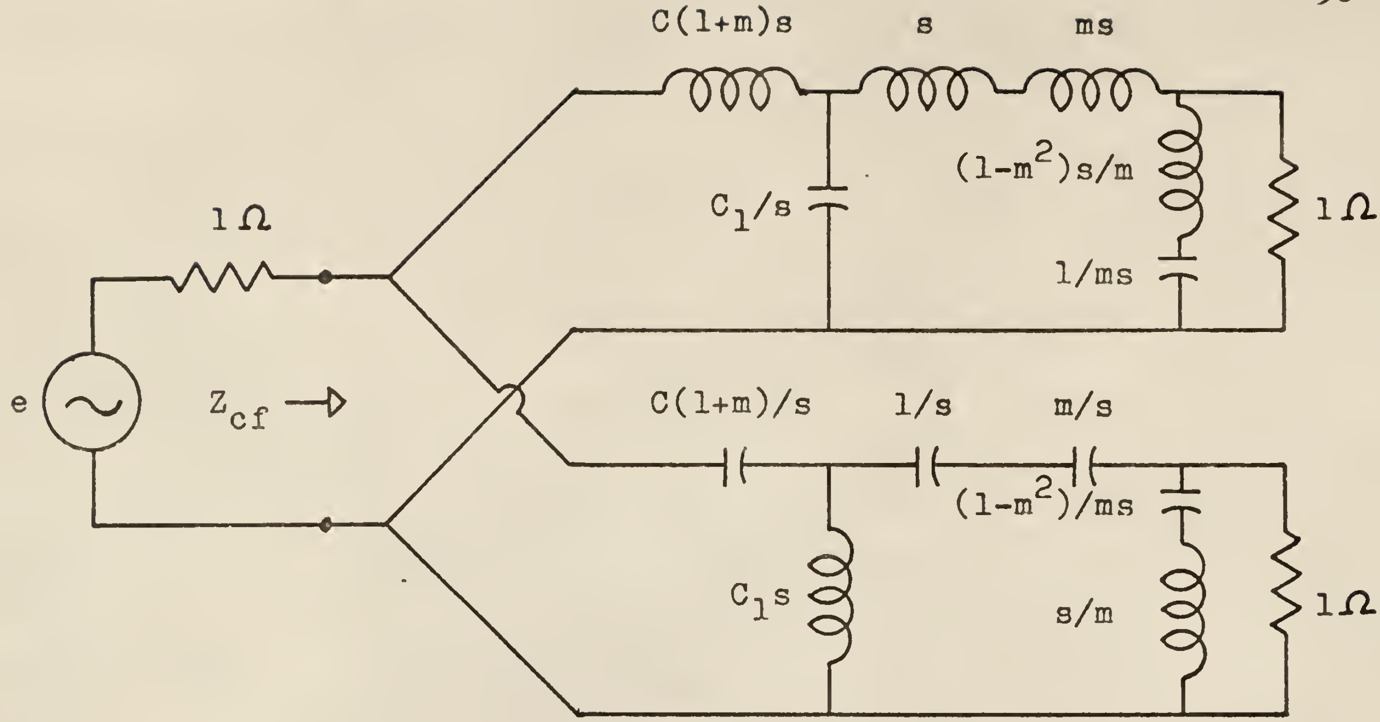


Fig. 45. Type 7 degenerate complementary Zobel filter with parameters C, C_1 and m.

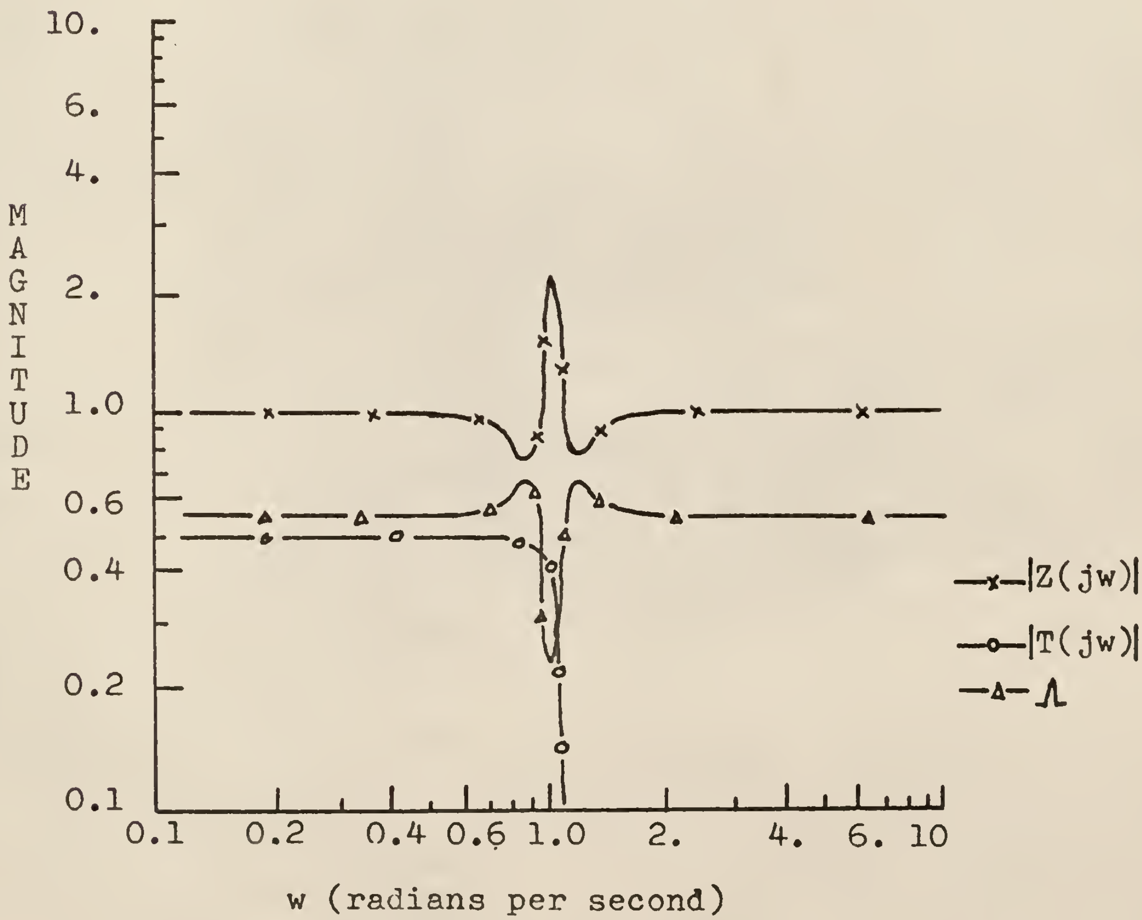


Fig. 46. Type 7 frequency response characteristics.

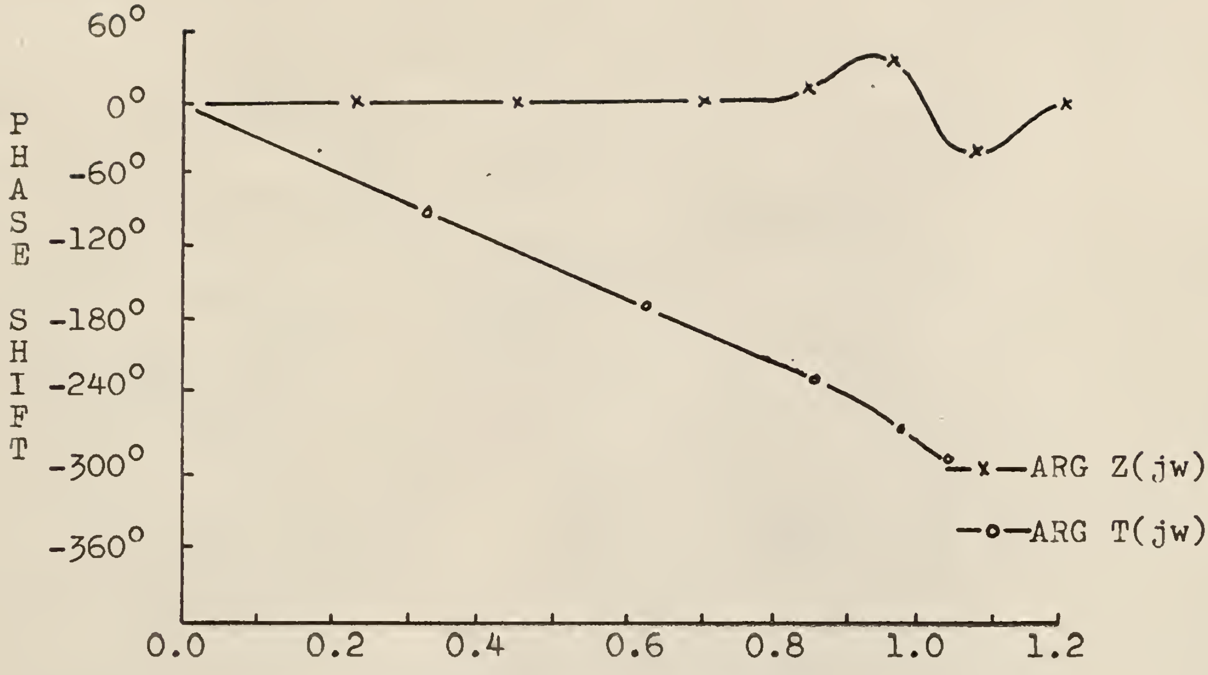


Fig. 47. Type 7 frequency response characteristics.

assertions as they have not deteriorated from those of Type 6, but have actually been improved.

CONCLUSIONS

Using the procedures of Zobel, Bode, Guillemin, Norton, Rowlands, and Fritzemeyer the Zobel process, aidentity algorithm, and impedance elision are defined, codified, and verified as methods of improving the frequency response characteristics of constant-k and constant-k complementary filters. It is shown that these methods not only improve the ADPI or interactance, but also improve the VTF and linearize the phase shift.

In the course of this verification several related topics of interest were studied and are included in the appendices. These topics are the FORGO program for evaluating the coefficients of a pseudo-generalized "ladder" VTF, the FORTRAN program for coefficient evaluation of the product of two polynomials with algebraic coefficients, the proof of the complementary filter ADPI coefficient symmetry, and the ADPI parameter evaluation for a Type 7 filter.

ACKNOWLEDGMENTS

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He acknowledges the help of Dr. Benton D. Weathers and Mr. Larry B. Hofman in computer programming.

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APPENDIX A

Impedance Elision. Webster's definition of elision: A cutting off, especially of a vowel, for the sake of meter or euphony; the dropping or partial pronounciation of a final vowel before an initial vowel in the next word. Fritzemeyer (1) used impedance elision without naming it. Given the complementary filter configuration of Fig. 48, it was noted that if the two shunt arms nearest the input terminals were removed that the effects of each could be substituted by the first series component and the next shunt arm element. Or when the shunt arms, $Z_{L2\&3}$ and $Z_{H2\&3}$, are removed as shown in Fig. 49, Z_{H1} substitutes for Z_{L3}, Z_{H4} substitutes for Z_{L2}, Z_{L1} substitutes for Z_{H3}, and Z_{L4} substitutes for Z_{H2}. This process slurs the effect that would be present if the two shunt arms were not removed and thus improves the complementary filter characteristics. This process will only be applied to the complementary Zobel filter configuration.

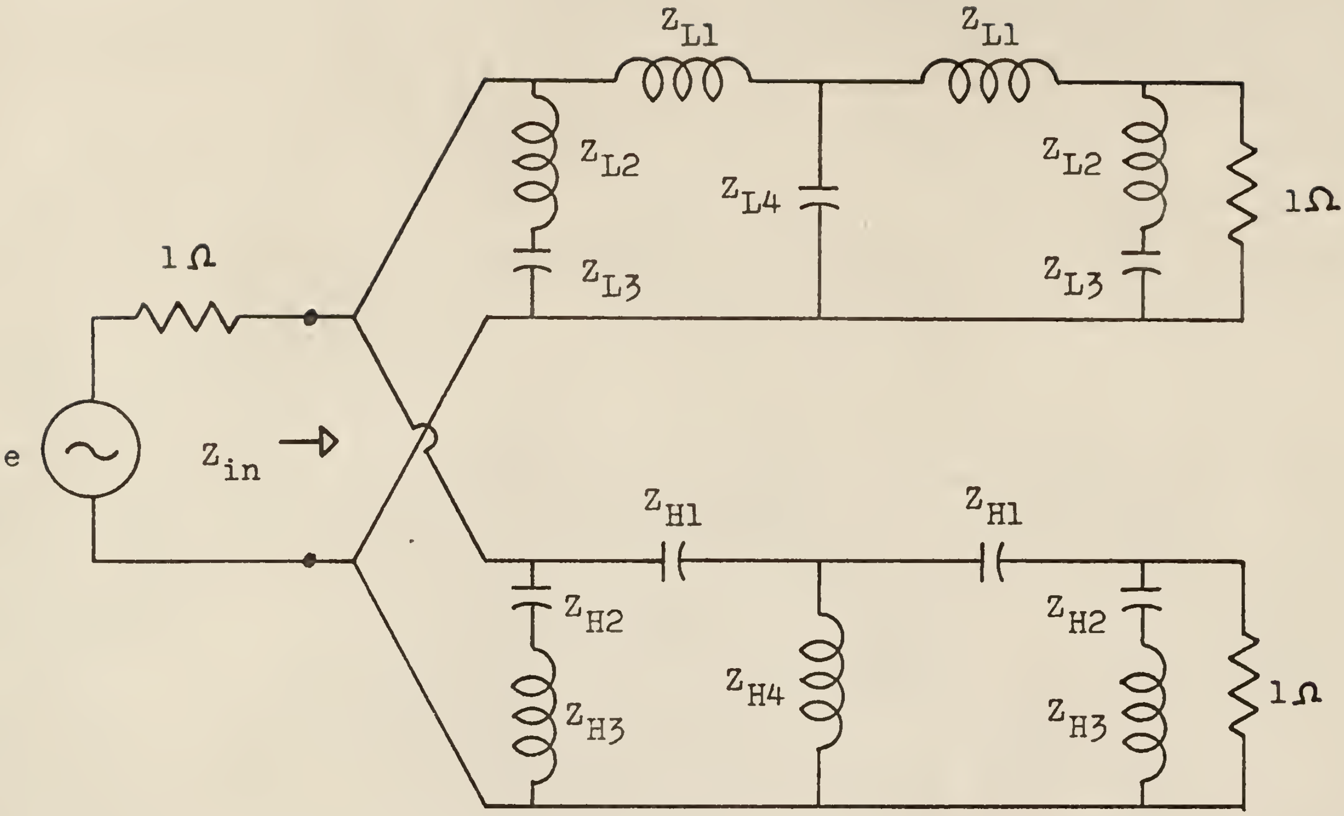

Fig. 48. Complementary filter configuration.

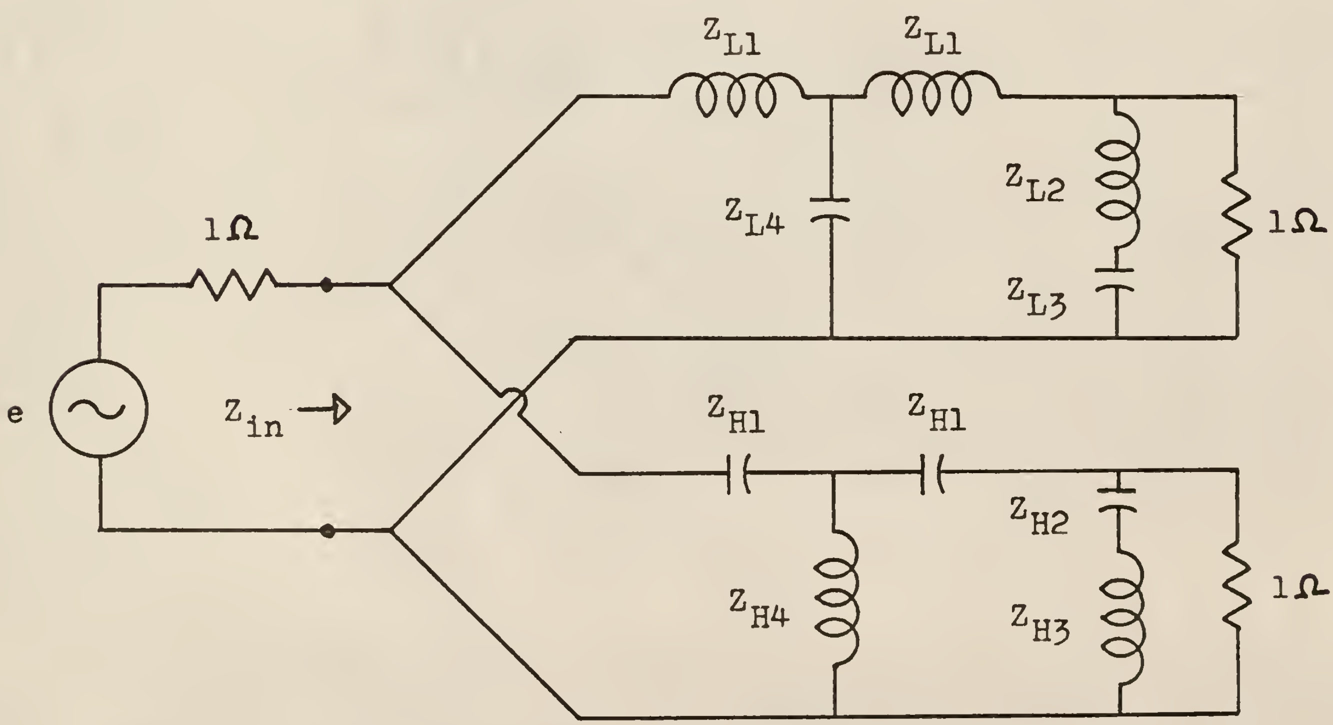

Fig. 49. Complementary degenerate filter configuration.

APPENDIX B

<u>Aidentity Algorithm.</u> Using the principle of King's (2) approximate identity on the ADPI of a filter is the aidentity algorithm.

Given the constant-1 filter configuration of Fig. 50, the aidentity algorithm is applied to the ADPI of equation (27). This process is to equate as many successive coefficients of the corresponding powers of s in the numerator

$$Z_{in} = \frac{A_o+A_1s+A_2s^2+A_3s^3}{B_o+B_1s+B_2s^2+B_3s^3} \tag{27}$$

where

$$\begin{aligned} A_o &= 1 & B_o &= 1 \\ A_1 &= 2 & B_1 &= 2 \\ A_2 &= 2\lambda & B_2 &= 2(2-\lambda) \\ A_3 &= 2\lambda(2-\lambda) & B_3 &= 0 \end{aligned} \tag{28}$$

cients of the corresponding powers of s in the numerator and denominator as is possible. This has $A_o=B_o$, $A_1=B_1$, and $A_2=B_2$. The result of A_2 equated to B_2 is $\lambda=1$. When this value of λ is substituted in $A_3=B_3$, the result $1\neq 0$ implies that A_3 and B_3 cannot be equated if A_2 and B_2 are. Since the process has to use successive coefficients, $\lambda=1$ will be used and the resultant ADPI will be a third order ADPI. However this ADPI will be the same as would be obtained if the constant-1 filter of Fig. 51 was used. Thus this filter aidentity order cannot be improved by the aidentity algorithm.

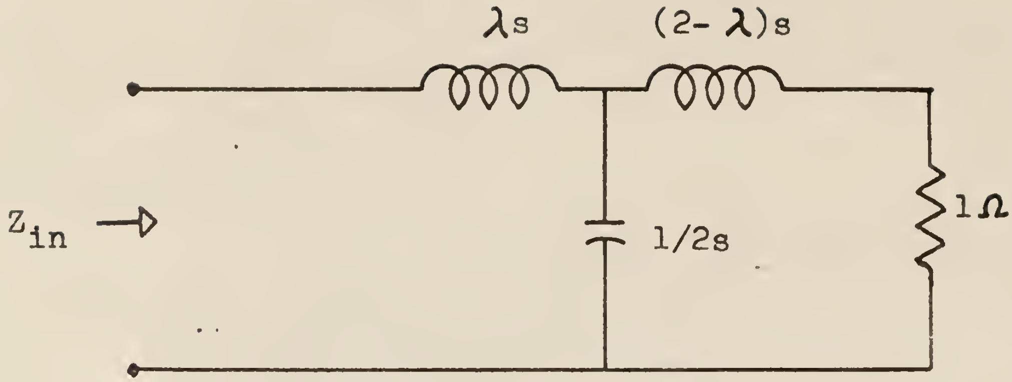

Fig. 50. Low-pass filter.

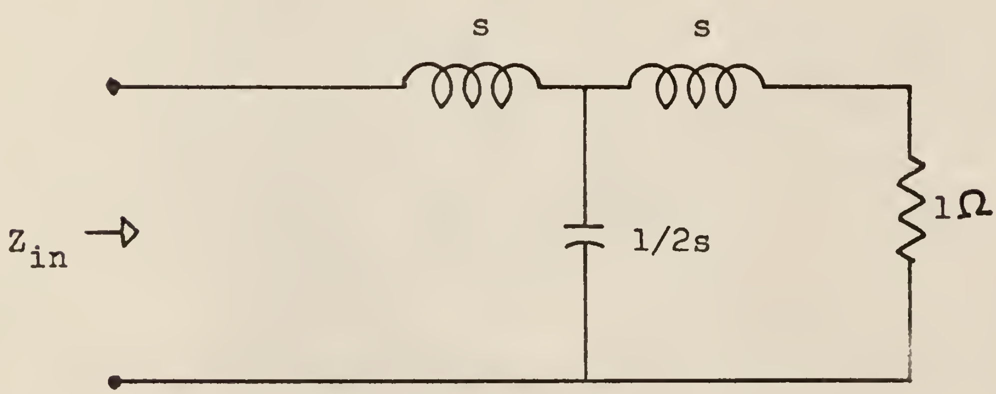

Fig. 51. Constant-1 filter.

APPENDIX C

Interactance. Interactance (1) is a means of comparing the power available at the input terminals of a filter, or a fan-out filter configuration of two or more filters, with that which would be available if all of the filters were purely resistive. Therefore a constant interactance frequency response would indicate a network that was resistive in nature. The ADPI phase shift frequency response would give the same information, but computational wise it is more difficult. Interactance is more realistic than characteristic impedance in fan-out filter configurations because of the full range of frequencies that are being encountered by this type of network. Characteristic impedance only has meaning at a single frequency.

In the calculation of interactance, $\Lambda = P_f/P_r$, P_f and P_r are calculated for e being a peak value of the sinusoidal voltage source of Fig. 52, $E_{eff}=|e|^2/\sqrt{2}$, and the source resistance, R_o, is not matched to the filter driving point impedance. Referring to Fig. 52, P_r is the power dissipated in the source resistance when a pure resistance of one ohm replaces each filter in the fan-out network and a voltage source, $-e/n$, is placed in series with each of these resistances. Then from Fig. 52 using Millman's theorem, $R_o=R_1= \ldots =R_n=1\Omega$, $Y_o=1/R_o$, $Y_1=Y_2= \ldots =Y_n=1\mho$,

$$e_{or} = \frac{e(1) + \left[(-e/n)(1)\right]n}{1 + (1)n} \tag{29}$$

$$e_{or} = 0 \tag{30}$$

$$e_{rr} = e - e_{or} = e \tag{31}$$

$$E_{rreff} = |e_{rr}|/\sqrt{2} \tag{32}$$

$$P_r = |e_{rr}|^2/2R_o \tag{33}$$

$$\therefore P_r = |e|^2/2 \tag{34}$$

Referring to Fig. 53, P_f is the power dissipated in the source resistance when a voltage source, $-e/n$, is placed in series with each fan-out filter. Then from Fig. 53 using Millman's theorem, $R_o = 1\Omega$, $Y_o = 1/R_o$,

$$e_{of} = \frac{eY_o + (-e/n)\left[Y_1 + Y_2 + \ldots + Y_n\right]}{Y_o + Y_1 + Y_2 + \ldots + Y_n} \tag{35}$$

$$e_{rf} = e - e_{of} = e\left[\frac{(n+1)(Y_1+Y_2+\ldots+Y_n)}{n(Y_o+Y_1+\ldots+Y_n)}\right] \tag{36}$$

$$E_{rfeff} = \frac{|e_{rf}|}{\sqrt{2}} \tag{37}$$

$$P_f = \frac{|e_{rf}|^2 Y_o}{2} \tag{38}$$

$$\therefore P_f = \frac{|e|^2}{2}\left|\frac{(n+1)(Y_1+Y_2+\ldots+Y_n)}{n(1+Y_1+Y_2+\ldots+Y_n)}\right|^2 \tag{39}$$

The interactance is now the ratio of P_f to P_r as given in equation (40).

$$\Lambda = \frac{P_f}{P_r} = \left|\frac{(n+1)(Y_1+Y_2+\ldots+Y_n)}{n(1+Y_1+Y_2+\ldots+Y_n)}\right|^2 \tag{40}$$

The following observations should be made concerning the computational techniques just given:

1. e, e_{rr}, e_{rf}, e_{or}, and e_{of} are peak values of sinusoidal voltages and used for ease of computation.
2. $(-e/n)$ voltages were added for ease of computation.
3. One ohm resistors and impedances were assumed for ease of computation since filters were normalized to impedance level of one ohm.
4. The interactance for filters at different impedance levels can be determined, but this just confuses the issue.

As an example to show the interactance calculations, the constant-l filter is used as shown in Fig. 51. The ADPI is given in equation (41). In this example n=1 as

there is only one filter and Y_1 is the inverse of the ADPI as shown in equation (42). Because this filter is normalized to one ohm, Y_o will be equal to one mho. Using these values in equation (40) results in equation (43) and this interactance response curve is given in Fig. 20.

$$Z_{in} = \frac{1+2s+2s^2+2s^3}{1+2s+2s^2} \tag{41}$$

$$Y_1 = \frac{1+2s+2s^2}{1+2s+2s^2+2s^3} \tag{42}$$

Since $Y_o = 1\mho$ and $n=1$, then

$$\Lambda = \left. \left| \frac{1+2s+2s^2}{1+2s+2s^2+s^3} \right|^2 \right|_{s=jw} = \frac{1+4w^4}{1+w^6} \tag{43}$$

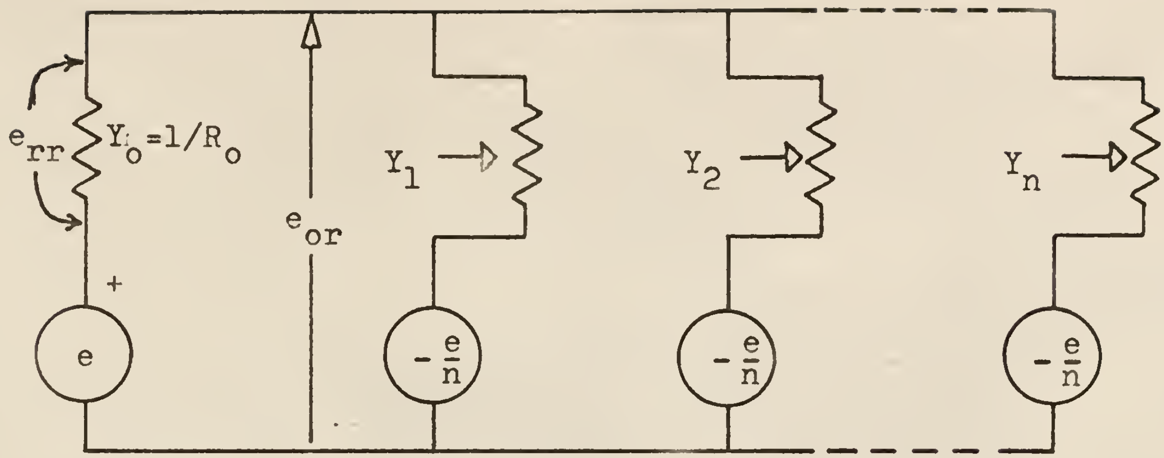

Fig. 52. The fan-out resistive network used for the determination of P_r for interactance.

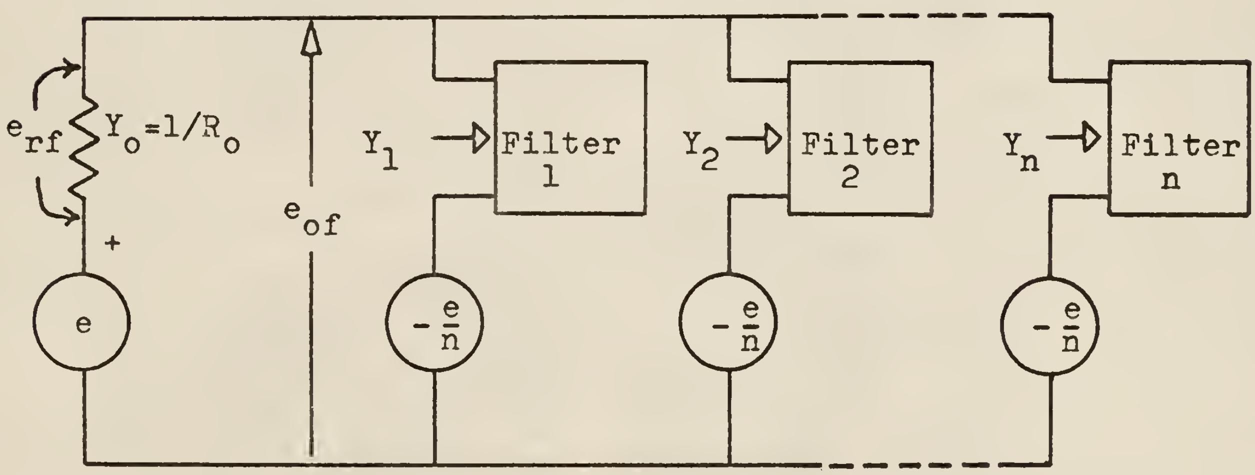

Fig. 53. The fan-out filter network used for the determination of P_f for interactance.

APPENDIX D

FORGO Programs for Interactance Calculations. To make possible the computation of the interactance versus frequency graph, the FORGO digital computer program shown on pages 65 through 69 was written and used. The input and output data for each of the previously discussed filter configurations are given on the pages following the computer program.

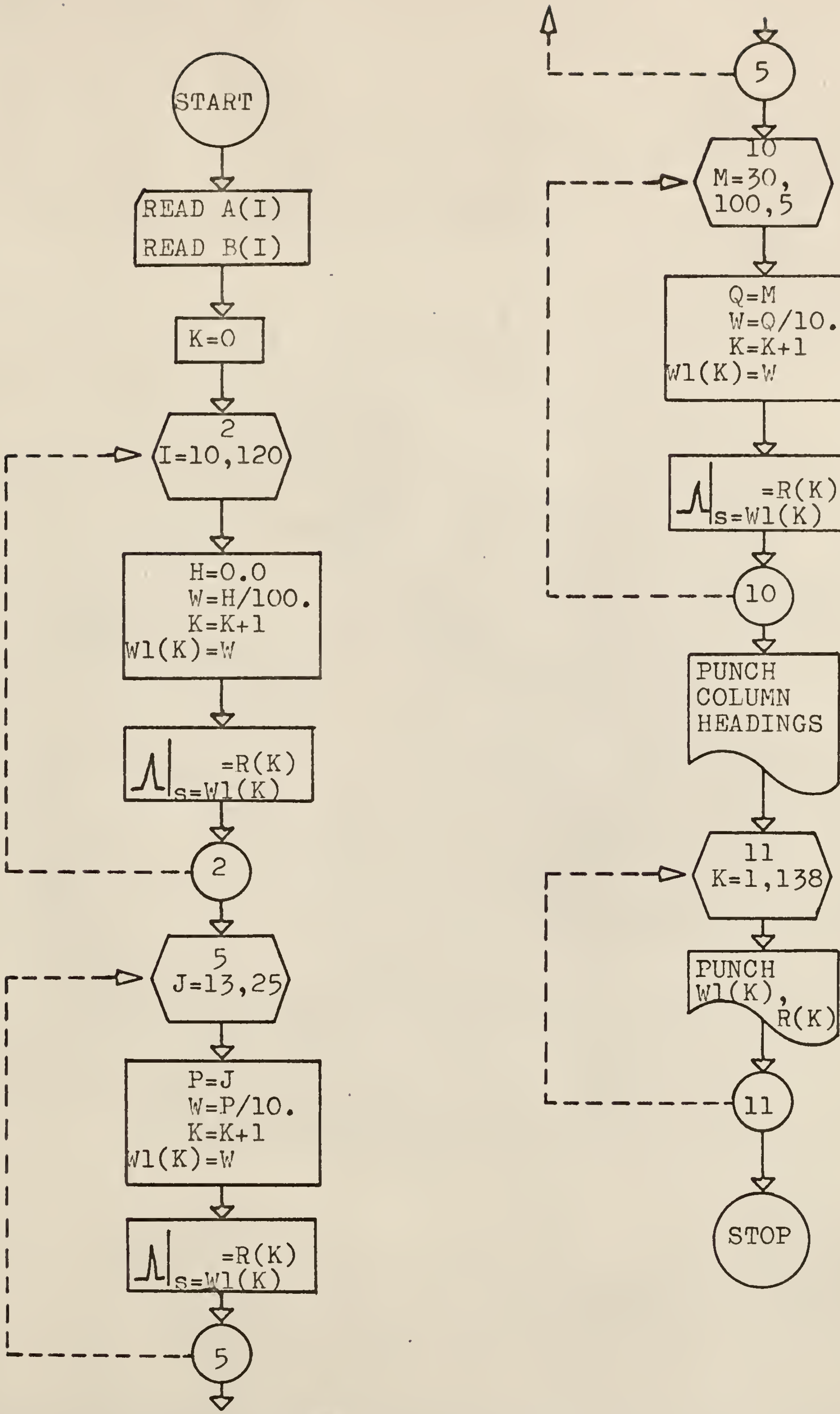

Fig. 54. Block diagram of interactance computer program.

```
C   C  INTERACTANCE VS. FREQUENCY, SINGLE FILTERS
       DIMENSION A(10),B(10),W1(140),R(140)
     3 READ,A0,(A(I),I=1,10)
       READ,B0,(B(N),N=1,10)
       K=0
       DO 2 I=10,120
       H=I
       W=H/100.
       K=K+1
       W1(K)=W
       D=(B0-B(2)*W**2+B(4)*W**4-B(6)*W**6+B(8)*W**8-B(10)*W**10)**2
       E=(B(1)*W-B(3)*W**3+B(5)*W**5-B(7)*W**7+B(9)*W**9)**2
       F1=A0+B0-(A(2)+B(2))*W**2+(A(4)+B(4))*W**4-(A(6)+B(6))*W**6
       F2=(A(8)+B(8))*W**8-(A(10)+B(10))*W**10
       F=(F1+F2)**2
       G1=(A(1)+B(1))*W-(A(3)+B(3))*W**3+(A(5)+B(5))*W**5
       G2=-(A(7)+B(7))*W**7+(A(9)+B(9))*W**9
       G=(G1+G2)**2
       R(K)=4.     *(D+E)/(F+G)
     2 CONTINUE
       DO 5 J=13,25
       P=J
       W=P/10.
       K=K+1
       W1(K)=W
       D=(B0-B(2)*W**2+B(4)*W**4-B(6)*W**6+B(8)*W**8-B(10)*W**10)**2
       E=(B(1)*W-B(3)*W**3+B(5)*W**5-B(7)*W**7+B(9)*W**9)**2
       F1=A0+B0-(A(2)+B(2))*W**2+(A(4)+B(4))*W**4-(A(6)+B(6))*W**6
       F2=(A(8)+B(8))*W**8-(A(10)+B(10))*W**10
       F=(F1+F2)**2
       G1=(A(1)+B(1))*W-(A(3)+B(3))*W**3+(A(5)+B(5))*W**5
       G2=-(A(7)+B(7))*W**7+(A(9)+B(9))*W**9
       G=(G1+G2)**2
       R(K)=4.     *(D+E)/(F+G)
     5 CONTINUE
       DO 10 M=30,100,5
       Q=M
       W=Q/10.
       K=K+1
       W1(K)=W
       D=(B0-B(2)*W**2+B(4)*W**4-B(6)*W**6+B(8)*W**8-B(10)*W**10)**2
       E=(B(1)*W-B(3)*W**3+B(5)*W**5-B(7)*W**7+B(9)*W**9)**2
       F1=A0+B0-(A(2)+B(2))*W**2+(A(4)+B(4))*W**4-(A(6)+B(6))*W**6
       F2=(A(8)+B(8))*W**8-(A(10)+B(10))*W**10
       F=(F1+F2)**2
       G1=(A(1)+B(1))*W-(A(3)+B(3))*W**3+(A(5)+B(5))*W**5
       G2=-(A(7)+B(7))*W**7+(A(9)+B(9))*W**9
       G=(G1+G2)**2
       R(K)=4.     *(D+E)/(F+G)
    10 CONTINUE
       PUNCH 4
```

```
      DO 11 K=1,45
      PUNCH 7,W1(K),R(K),W1(K+47),R(K+47),W1(K+94),R(K+94)
   11 CONTINUE
      DO 12 K=46,47
      PUNCH 8,W1(K),R(K),W1(K+47),R(K+47)
   12 CONTINUE
      GO TO 3
    4 FORMAT(/8X,1HW,9X,1HR,2(10X,1HW,9X,1HR)//)
    7 FORMAT(3(5X,F5.2,3X,F8.5))
    8 FORMAT(2(5X,F5.2,3X,F8.5))
      END
```

```
C  C  INTERACTANCE VS. FREQUENCY, COMPLEMENTARY FILTERS
      DIMENSION A(10),B(10),W1(140),R(140)
    3 READ,A0,(A(I),I=1,10)
      READ,B0,(B(N),N=1,10)
      K=0
      DO 2 I=10,120
      H=I
      W=H/100.
      K=K+1
      W1(K)=W
      D=(B0-B(2)*W**2+B(4)*W**4-B(6)*W**6+B(8)*W**8-B(10)*W**10)**2
      E=(B(1)*W-B(3)*W**3+B(5)*W**5-B(7)*W**7+B(9)*W**9)**2
      F1=A0+B0-(A(2)+B(2))*W**2+(A(4)+B(4))*W**4-(A(6)+B(6))*W**6
      F2=(A(8)+B(8))*W**8-(A(10)+B(10))*W**10
      F=(F1+F2)**2
      G1=(A(1)+B(1))*W-(A(3)+B(3))*W**3+(A(5)+B(5))*W**5
      G2=-(A(7)+B(7))*W**7+(A(9)+B(9))*W**9
      G=(G1+G2)**2
      R(K)=(9./4.)*(D+E)/(F+G)
    2 CONTINUE
      DO 5 J=13,25
      P=J
      W=P/10.
      K=K+1
      W1(K)=W
      D=(B0-B(2)*W**2+B(4)*W**4-B(6)*W**6+B(8)*W**8-B(10)*W**10)**2
      E=(B(1)*W-B(3)*W**3+B(5)*W**5-B(7)*W**7+B(9)*W**9)**2
      F1=A0+B0-(A(2)+B(2))*W**2+(A(4)+B(4))*W**4-(A(6)+B(6))*W**6
      F2=(A(8)+B(8))*W**8-(A(10)+B(10))*W**10
      F=(F1+F2)**2
      G1=(A(1)+B(1))*W-(A(3)+B(3))*W**3+(A(5)+B(5))*W**5
      G2=-(A(7)+B(7))*W**7+(A(9)+B(9))*W**9
      G=(G1+G2)**2
      R(K)=(9./4.)*(D+E)/(F+G)
    5 CONTINUE
      DO 10 M=30,100,5
      Q=M
      W=Q/10.
      K=K+1
      W1(K)=W
      D=(B0-B(2)*W**2+B(4)*W**4-B(6)*W**6+B(8)*W**8-B(10)*W**10)**2
      E=(B(1)*W-B(3)*W**3+B(5)*W**5-B(7)*W**7+B(9)*W**9)**2
      F1=A0+B0-(A(2)+B(2))*W**2+(A(4)+B(4))*W**4-(A(6)+B(6))*W**6
      F2=(A(8)+B(8))*W**8-(A(10)+B(10))*W**10
      F=(F1+F2)**2
      G1=(A(1)+B(1))*W-(A(3)+B(3))*W**3+(A(5)+B(5))*W**5
      G2=-(A(7)+B(7))*W**7+(A(9)+B(9))*W**9
      G=(G1+G2)**2
      R(K)=(9./4.)*(D+E)/(F+G)
   10 CONTINUE
      PUNCH 4
```

```
      DO 11 K=1,45
      PUNCH 7,W1(K),R(K),W1(K+47),R(K+47),W1(K+94),R(K+94)
   11 CONTINUE
      DO 12 K=46,47
      PUNCH 8,W1(K),R(K),W1(K+47),R(K+47)
   12 CONTINUE
      GO TO 3
    4 FORMAT(/8X,1HW,9X,1HR,2(10X,1HW,9X,1HR)//)
    7 FORMAT(3(5X,F5.2,3X,F8.5))
    8 FORMAT(2(5X,F5.2,3X,F8.5))
      END
```

```
INTERACTANCE RESPONSE INPUT DATA, ADPI COEFF., CONSTANT-1 FILTER
1.  2.  2.  2.  .0  .0  .0  .0  .0  .0  .0
1.  2.  2.  .0  .0  .0  .0  .0  .0  .0  .0

INTERACTANCE RESPONSE INPUT DATA, ADPI COEFF., ZOBEL FILTER, M=0.6
1.  3.84  5.76  11.63  6.32  9.1  1.95  2.07  .0  .0  .0
1.  3.2  8.06  6.64  10.81  3.07  3.26  .0  .0  .0  .0

INTERACTANCE RESPONSE INPUT DATA, ADPI COEFF., ZOBEL FILTER, M=0.707
1.  3.414  6.828  9.246  8.992  6.682  2.914  1.457  .0  .0  .0
1.  3.414  6.828  9.242  8.992  5.828  2.914  .0  .0  .0  .0

INTERACTANCE RESPONSE INPUT DATA, ADPI COEFF., TYPE 1 FILTER
1.  3.  5.  6.5  5.  3.  1.  .0  .0  .0  .0
1.  4.  8.  9.  8.  4.  1.  .0  .0  .0  .0

INTERACTANCE RESPONSE INPUT DATA, ADPI COEFF., TYPE 2 FILTER
3.24  11.7  21.55  28.79  21.55  11.7  3.24  .0  .0  .0  .0
3.24  11.7  22.8  25.42  22.8  11.7  3.24  .0  .0  .0  .0

INTERACTANCE RESPONSE INPUT DATA, ADPI COEFF., TYPE 3 FILTER
3.73246  13.92954  26.85862  36.3263  26.85862  13.92954  3.73246  .0
.0  .0  .0
3.73246  13.92954  26.8614  29.859  26.8614  13.92954  3.73246  .0  .0
.0  .0

INTERACTANCE RESPONSE INPUT DATA, ADPI COEFF., TYPE 4 FILTER
.9975  3.99  7.98  9.95  7.98  3.99  .9975  .0  .0  .0  .0
.9975  3.99  7.98  9.95  7.98  3.99  .9975  .0  .0  .0  .0

INTERACTANCE RESPONSE INPUT DATA, ADPI COEFF., TYPE 6 FILTER
3.23607 15.70820 43.12461 83.33938 119.37381 135.99185 119.37381
83.33938 43.12461 15.70820 3.23607        3.23607 15.70820 43.12461
81.48529 118.37383 131.8459 118.37383 81.48529 43.12461 15.70820 3.23607

INTERACTANCE RESPONSE INPUT DATA, ADPI COEFF., TYPE 7 FILTER
      .68140        3.63470        10.42514        20.45234 29.90444
    33.93411        29.904444      20.45234        10.42514 3.6347
      .68140
      .68140        3.63470        10.42514        20.45234 29.93704
    33.77306        29.93704       20.45234        10.42514 3.6347
      .68140
```

C C INTERACTANCE VS. FREQUENCY, OUTPUT DATA, CONSTANT-1 FILTER, FIG.19

W	R	W	R	W	R
.10	1.00040	.57	1.37508	1.04	2.50712
.11	1.00058	.58	1.39939	1.05	2.50504
.12	1.00083	.59	1.42460	1.06	2.50149
.13	1.00114	.60	1.45072	1.07	2.49654
.14	1.00153	.61	1.47770	1.08	2.49025
.15	1.00201	.62	1.50554	1.09	2.48266
.16	1.00260	.63	1.53420	1.10	2.47384
.17	1.00332	.64	1.56364	1.11	2.46385
.18	1.00416	.65	1.59382	1.12	2.45276
.19	1.00517	.66	1.62470	1.13	2.44063
.20	1.00634	.67	1.65623	1.14	2.42751
.21	1.00769	.68	1.68833	1.15	2.41349
.22	1.00926	.69	1.72096	1.16	2.39861
.23	1.01104	.70	1.75404	1.17	2.38293
.24	1.01308	.71	1.78749	1.18	2.36653
.25	1.01538	.72	1.82123	1.19	2.34946
.26	1.01796	.73	1.85518	1.20	2.33177
.27	1.02086	.74	1.88924	1.30	2.13228
.28	1.02409	.75	1.92332	1.40	1.91879
.29	1.02768	.76	1.95731	1.50	1.71501
.30	1.03165	.77	1.99113	1.60	1.53086
.31	1.03602	.78	2.02465	1.70	1.36880
.32	1.04083	.79	2.05778	1.80	1.22787
.33	1.04609	.80	2.09041	1.90	1.10578
.34	1.05183	.81	2.12243	2.00	1.00000
.35	1.05808	.82	2.15374	2.10	.90810
.36	1.06487	.83	2.18422	2.20	.82796
.37	1.07222	.84	2.21379	2.30	.75778
.38	1.08015	.85	2.24233	2.40	.69604
.39	1.08871	.86	2.26976	2.50	.64147
.40	1.09790	.87	2.29599	3.00	.44521
.41	1.10777	.88	2.32093	3.50	.32690
.42	1.11833	.89	2.34451	4.00	.25018
.43	1.12961	.90	2.36666	4.50	.19763
.44	1.14164	.91	2.38732	5.00	.16005
.45	1.15444	.92	2.40642	5.50	.13226
.46	1.16803	.93	2.42394	6.00	.11113
.47	1.18244	.94	2.43983	6.50	.09469
.48	1.19769	.95	2.45406	7.00	.08164
.49	1.21379	.96	2.46662	7.50	.07112
.50	1.23077	.97	2.47749	8.00	.06250
.51	1.24864	.98	2.48667	8.50	.05537
.52	1.26741	.99	2.49417	9.00	.04938
.53	1.28709	1.00	2.50000	9.50	.04432
.54	1.30770	1.01	2.50418	10.00	.04000
.55	1.32923	1.02	2.50674		
.56	1.35169	1.03	2.50770		

C C INTERACTANCE VS. FREQUENCY, OUTPUT DATA, ZOBEL FILTER, FIG.22

W	R	W	R	W	R
.10	.95915	.57	1.53551	1.04	2.07124
.11	.95148	.58	1.56422	1.05	2.28933
.12	.94343	.59	1.59062	1.06	2.49755
.13	.93509	.60	1.61434	1.07	2.69356
.14	.92657	.61	1.63501	1.08	2.87578
.15	.91796	.62	1.65225	1.09	3.04322
.16	.90936	.63	1.66568	1.10	3.19545
.17	.90089	.64	1.67493	1.11	3.33245
.18	.89267	.65	1.67962	1.12	3.45456
.19	.88481	.66	1.67938	1.13	3.56234
.20	.87742	.67	1.67388	1.14	3.65651
.21	.87063	.68	1.66275	1.15	3.73792
.22	.86455	.69	1.64569	1.16	3.80745
.23	.85931	.70	1.62240	1.17	3.86601
.24	.85503	.71	1.59262	1.18	3.91448
.25	.85182	.72	1.55611	1.19	3.95374
.26	.84978	.73	1.51272	1.20	3.98459
.27	.84905	.74	1.46231	1.30	3.97393
.28	.84970	.75	1.40485	1.40	3.69301
.29	.85185	.76	1.34039	1.50	3.41650
.30	.85559	.77	1.26910	1.60	3.16696
.31	.86099	.78	1.19126	1.70	2.91334
.32	.86814	.79	1.10735	1.80	2.66868
.33	.87709	.80	1.01802	1.90	2.44280
.34	.88790	.81	.92415	2.00	2.23846
.35	.90650	.82	.82691	2.10	2.05518
.36	.91524	.83	.72775	2.20	1.89129
.37	.93181	.84	.62847	2.30	1.74476
.38	.95032	.85	.53123	2.40	1.61360
.39	.97075	.86	.43857	2.50	1.49598
.40	.99307	.87	.35338	3.00	1.05989
.41	1.01722	.88	.27889	3.50	.78745
.42	1.04313	.89	.21856	4.00	.60709
.43	1.07071	.90	.17595	4.50	.48190
.44	1.09986	.91	.15456	5.00	.39160
.45	1.13044	.92	.15761	5.50	.32441
.46	1.16231	.93	.18774	6.00	.27308
.47	1.19529	.94	.24682	6.50	.23300
.48	1.22921	.95	.33567	7.00	.20112
.49	1.26384	.96	.45393	7.50	.17535
.50	1.29896	.97	.59998	8.00	.15423
.51	1.33432	.98	.77100	8.50	.13670
.52	1.36966	.99	.96315	9.00	.12199
.53	1.40469	1.00	1.17181	9.50	.10953
.54	1.43911	1.01	1.39195	10.00	.09889
.55	1.47261	1.02	1.61843		
.56	1.50486	1.03	1.84634		

C C INTERACTANCE VS. FREQUENCY, OUTPUT DATA, ZOBEL FILTER, FIG.25

W	R	W	R	W	R
.10	1.00000	.57	.97338	1.04	1.09083
.11	1.00000	.58	.97270	1.05	1.29403
.12	.99999	.59	.97222	1.06	1.49718
.13	.99999	.60	.97197	1.07	1.69594
.14	.99998	.61	.97197	1.08	1.88726
.15	.99998	.62	.97226	1.09	2.06919
.16	.99996	.63	.97284	1.10	2.24058
.17	.99995	.64	.97376	1.11	2.40095
.18	.99992	.65	.97501	1.12	2.55023
.19	.99989	.66	.97660	1.13	2.68868
.20	.99985	.67	.97852	1.14	2.81672
.21	.99980	.68	.98076	1.15	2.93487
.22	.99974	.69	.98328	1.16	3.04372
.23	.99966	.70	.98601	1.17	3.14385
.24	.99956	.71	.98888	1.18	3.23583
.25	.99945	.72	.99178	1.19	3.32023
.26	.99930	.73	.99454	1.20	3.39755
.27	.99914	.74	.99699	1.30	3.87360
.28	.99894	.75	.99888	1.40	4.00449
.29	.99871	.76	.99991	1.50	3.95467
.30	.99844	.77	.99974	1.60	3.81106
.31	.99812	.78	.99793	1.70	3.62201
.32	.99777	.79	.99398	1.80	3.41476
.33	.99736	.80	.98730	1.90	3.20462
.34	.99690	.81	.97723	2.00	2.99998
.35	.99639	.82	.96301	2.10	2.80528
.36	.99582	.83	.94381	2.20	2.62260
.37	.99518	.84	.91878	2.30	2.45264
.38	.99448	.35	.88702	2.40	2.29536
.39	.99372	.86	.84770	2.50	2.15026
.40	.99288	.87	.80016	3.00	1.58021
.41	.99198	.88	.74399	3.50	1.19950
.42	.99101	.89	.67930	4.00	.93751
.43	.98998	.90	.60690	4.50	.75112
.44	.98888	.91	.52860	5.00	.61441
.45	.98773	.92	.44748	5.50	.51145
.46	.98652	.93	.36809	6.00	.43211
.47	.98527	.94	.29641	6.50	.36974
.48	.98399	.95	.23963	7.00	.31987
.49	.98268	.96	.20537	7.50	.27939
.50	.98136	.97	.20077	8.00	.24610
.51	.98004	.98	.23117	8.50	.21839
.52	.97875	.99	.29921	9.00	.19509
.53	.97750	1.00	.40424	9.50	.17532
.54	.97631	1.01	.54250	10.00	.15840
.55	.97521	1.02	.70792		
.56	.97423	1.03	.89319		

C C INTERACTANCE VS. FREQUENCY, OUTPUT DATA, TYPE 1 FILTER, FIG.28

W	R	W	R	W	R
.10	.56714	.57	.94451	1.04	1.02592
.11	.56822	.58	.95993	1.05	1.03851
.12	.56944	.59	.97535	1.06	1.05237
.13	.57082	.60	.99071	1.07	1.06693
.14	.57236	.61	1.00596	1.08	1.08169
.15	.57408	.62	1.02105	1.09	1.09623
.16	.57599	.63	1.03594	1.10	1.11021
.17	.57810	.64	1.05057	1.11	1.12339
.18	.58044	.65	1.06489	1.12	1.13560
.19	.58300	.66	1.07884	1.13	1.14672
.20	.58581	.67	1.09236	1.14	1.15672
.21	.58888	.68	1.10540	1.15	1.16555
.22	.59222	.69	1.11788	1.16	1.17324
.23	.59585	.70	1.12976	1.17	1.17982
.24	.59979	.71	1.14095	1.18	1.18532
.25	.60404	.72	1.15140	1.19	1.18982
.26	.60863	.73	1.16102	1.20	1.19336
.27	.61357	.74	1.16976	1.30	1.18944
.28	.61886	.75	1.17752	1.40	1.14552
.29	.62453	.76	1.18424	1.50	1.08791
.30	.63058	.77	1.18983	1.60	1.02853
.31	.63703	.78	1.19420	1.70	.97263
.32	.64389	.79	1.19727	1.80	.92229
.33	.65116	.80	1.19894	1.90	.87800
.34	.65885	.81	1.19914	2.00	.83954
.35	.66697	.82	1.19777	2.10	.80638
.36	.67553	.83	1.19475	2.20	.77787
.37	.68[illegible]52	.84	1.19001	2.30	.75337
.38	.69396	.85	1.18350	2.40	.73230
.39	.70384	.86	1.17519	2.50	.71415
.40	.71415	.87	1.16507	3.00	.65367
.41	.72490	.88	1.15322	3.50	.62205
.42	.73608	.89	1.13973	4.00	.60404
.43	.74768	.90	1.12479	4.50	.59300
.44	.75969	.91	1.10871	5.00	.58581
.45	.77210	.92	1.09185	5.50	.58088
.46	.78489	.93	1.07471	6.00	.57738
.47	.79805	.94	1.05789	6.50	.57479
.48	.81156	.95	1.04206	7.00	.57283
.49	.82540	.96	1.02791	7.50	.57131
.50	.83954	.97	1.01612	8.00	.57011
.51	.85396	.98	1.00728	8.50	.56914
.52	.86862	.99	1.00183	9.00	.56835
.53	.88350	1.00	1.00000	9.50	.56769
.54	.89857	1.01	1.00179	10.00	.56714
.55	.91378	1.02	1.00700		
.56	.92911	1.03	1.01522		

C C INTERACTANCE VS. FREQUENCY, OUTPUT DATA, TYPE 2 FILTER, FIG.31

W	R	W	R	W	R
.10	.56069	.57	.69809	1.04	.19258
.11	.56041	.58	.70347	1.05	.20589
.12	.56013	.59	.70850	1.06	.22141
.13	.55986	.60	.71309	1.07	.23881
.14	.55963	.61	.71719	1.08	.25776
.15	.55943	.62	.72071	1.09	.27793
.16	.55928	.63	.72357	1.10	.29902
.17	.55919	.64	.72568	1.11	.32073
.18	.55917	.65	.72696	1.12	.34279
.19	.55923	.66	.72732	1.13	.36497
.20	.55939	.67	.72667	1.14	.38705
.21	.55966	.68	.72492	1.15	.40886
.22	.56005	.69	.72196	1.16	.43023
.23	.56057	.70	.71772	1.17	.45103
.24	.56124	.71	.71211	1.18	.47118
.25	.56207	.72	.70503	1.19	.49057
.26	.56307	.73	.69642	1.20	.50915
.27	.56425	.74	.68619	1.30	.64659
.28	.56563	.75	.67430	1.40	.70926
.29	.56721	.76	.66070	1.50	.72701
.30	.56901	.77	.64535	1.60	.72223
.31	.57103	.78	.62824	1.70	.70764
.32	.57329	.79	.60939	1.80	.68982
.33	.57579	.80	.58884	1.90	.67196
.34	.57854	.81	.56665	2.00	.65546
.35	.58155	.82	.54292	2.10	.64084
.36	.58481	.83	.51780	2.20	.62819
.37	.58834	.84	.49147	2.30	.61738
.38	.59212	.85	.46415	2.40	.60821
.39	.59617	.86	.43609	2.50	.60048
.40	.60048	.87	.40761	3.00	.57668
.41	.60504	.88	.37905	3.50	.56651
.42	.60984	.89	.35076	4.00	.56207
.43	.61488	.90	.32317	4.50	.56015
.44	.62015	.91	.29667	5.00	.55939
.45	.62563	.92	.27168	5.50	.55917
.46	.63130	.93	.24862	6.00	.55921
.47	.63715	.94	.22787	6.50	.55936
.48	.64314	.95	.20978	7.00	.55956
.49	.64925	.96	.19463	7.50	.55978
.50	.65546	.97	.18268	8.00	.55999
.51	.66172	.98	.17408	8.50	.56019
.52	.66800	.99	.16891	9.00	.56037
.53	.67426	1.00	.16720	9.50	.56054
.54	.68044	1.01	.16888	10.00	.56069
.55	.68652	1.02	.17381		
.56	.69242	1.03	.18179		

C C INTERACTANCE VS. FREQUENCY, OUTPUT DATA, TYPE 3 FILTER, FIG.34

W	R	W	R	W	R
.10	.56285	.57	.72474	1.04	.10384
.11	.56302	.58	.72820	1.05	.11537
.12	.56323	.59	.73102	1.06	.12895
.13	.56350	.60	.73309	1.07	.14434
.14	.56384	.61	.73436	1.08	.16132
.15	.56426	.62	.73471	1.09	.17965
.16	.56477	.63	.73408	1.10	.19909
.17	.56537	.64	.73237	1.11	.21943
.18	.56609	.65	.72948	1.12	.24044
.19	.56692	.66	.72535	1.13	.26194
.20	.56789	.67	.71988	1.14	.28373
.21	.56900	.68	.71299	1.15	.30563
.22	.57027	.69	.70462	1.16	.32751
.23	.57170	.70	.69470	1.17	.34922
.24	.57331	.71	.68318	1.18	.37064
.25	.57511	.72	.67000	1.19	.39167
.26	.57710	.73	.65515	1.20	.41222
.27	.57930	.74	.63861	1.30	.58068
.28	.58171	.75	.62038	1.40	.67774
.29	.58435	.76	.60048	1.50	.72186
.30	.58721	.77	.57897	1.60	.73453
.31	.59031	.78	.55592	1.70	.73057
.32	.59365	.79	.53143	1.80	.71875
.33	.59724	.80	.50561	1.90	.70390
.34	.60107	.81	.47863	2.00	.68855
.35	.60514	.82	.45067	2.10	.67392
.36	.60946	.83	.42193	2.20	.66057
.37	.61401	.84	.39266	2.30	.64866
.38	.61880	.85	.36312	2.40	.63818
.39	.62381	.86	.33359	2.50	.62904
.40	.62904	.87	.30438	3.00	.59849
.41	.63446	.88	.27578	3.50	.58319
.42	.64007	.89	.24813	4.00	.57511
.43	.64585	.90	.22173	4.50	.57057
.44	.65176	.91	.19691	5.00	.56789
.45	.65780	.92	.17394	5.50	.56623
.46	.66392	.93	.15310	6.00	.56516
.47	.67009	.94	.13464	6.50	.56444
.48	.67628	.95	.11876	7.00	.56395
.49	.68245	.96	.10561	7.50	.56361
.50	.68855	.97	.09533	8.00	.56336
.51	.69453	.98	.08798	8.50	.56317
.52	.70034	.99	.08359	9.00	.56304
.53	.70593	1.00	.08214	9.50	.56293
.54	.71123	1.01	.08356	10.00	.56285
.55	.71618	1.02	.08775		
.56	.72071	1.03	.09457		

C C INTERACTANCE VS. FREQUENCY, OUTPUT DATA, TYPE 6 FILTER, FIG.42

W	R	W	R	W	R
.10	.56263	.57	.58998	1.04	.30840
.11	.56270	.58	.58941	1.05	.35585
.12	.56277	.59	.58870	1.06	.40035
.13	.56287	.60	.58784	1.07	.43968
.14	.56300	.61	.58684	1.08	.47297
.15	.56315	.62	.58572	1.09	.50020
.16	.56333	.63	.58448	1.10	.52186
.17	.56355	.64	.58315	1.11	.53865
.18	.56380	.65	.58174	1.12	.55134
.19	.56409	.66	.58028	1.13	.56068
.20	.56442	.67	.57881	1.14	.56734
.21	.56480	.68	.57734	1.15	.57190
.22	.56523	.69	.57592	1.16	.57483
.23	.56570	.70	.57459	1.17	.57655
.24	.56623	.71	.57338	1.18	.57736
.25	.56680	.72	.57233	1.19	.57752
.26	.56743	.73	.57149	1.20	.57724
.27	.56812	.74	.57089	1.30	.57080
.28	.56885	.75	.57057	1.40	.57291
.29	.56964	.76	.57055	1.50	.57930
.30	.57048	.77	.57084	1.60	.58511
.31	.57137	.78	.57144	1.70	.58884
.32	.57230	.79	.57233	1.80	.59052
.33	.57327	.80	.57345	1.90	.59067
.34	.57429	.81	.57471	2.00	.58979
.35	.57534	.82	.57596	2.10	.58831
.36	.57641	.83	.57699	2.20	.58652
.37	.57751	.84	.57752	2.30	.58460
.38	.57862	.85	.57716	2.40	.58269
.39	.57974	.86	.57542	2.50	.58086
.40	.58086	.87	.57168	3.00	.57361
.41	.58196	.88	.56519	3.50	.56930
.42	.58305	.89	.55504	4.00	.56680
.43	.58411	.90	.54024	4.50	.56533
.44	.58513	.91	.51973	5.00	.56442
.45	.58610	.92	.49253	5.50	.56385
.46	.58700	.93	.45799	6.00	.56347
.47	.58784	.94	.41609	6.50	.56322
.48	.58859	.95	.36797	7.00	.56304
.49	.58924	.96	.31638	7.50	.56291
.50	.58979	.97	.26597	8.00	.56282
.51	.59023	.98	.22296	8.50	.56275
.52	.59054	.99	.19391	9.00	.56270
.53	.59072	1.00	.18366	9.50	.56266
.54	.59075	1.01	.19369	10.00	.56263
.55	.59065	1.02	.22152		
.56	.59039	1.03	.26180		

C C INTERACTANCE VS. FREQUENCY, OUTPUT DATA, TYPE 7 FILTER, FIG.45

W	R	W	R	W	R
.10	.56250	.57	.56118	1.04	.34493
.11	.56250	.58	.56204	1.05	.39497
.12	.56250	.59	.56311	1.06	.44441
.13	.56250	.60	.56442	1.07	.49061
.14	.56250	.61	.56598	1.08	.53197
.15	.56250	.62	.56784	1.09	.56775
.16	.56250	.63	.57001	1.10	.59785
.17	.56250	.64	.57253	1.11	.62249
.18	.56249	.65	.57542	1.12	.64216
.19	.56249	.66	.57871	1.13	.65742
.20	.56248	.67	.58243	1.14	.66885
.21	.56247	.68	.58660	1.15	.67701
.22	.56246	.69	.59123	1.16	.68241
.23	.56244	.70	.59634	1.17	.68555
.24	.56242	.71	.60193	1.18	.68681
.25	.56239	.72	.60800	1.19	.68655
.26	.56235	.73	.61453	1.20	.68508
.27	.56231	.74	.62148	1.30	.64361
.28	.56226	.75	.62880	1.40	.60448
.29	.56220	.76	.63641	1.50	.58114
.30	.56213	.77	.64422	1.60	.56888
.31	.56205	.78	.65207	1.70	.56291
.32	.56196	.79	.65979	1.80	.56025
.33	.56186	.80	.66716	1.90	.55927
.34	.56174	.81	.67389	2.00	.55911
.35	.56161	.82	.67965	2.10	.55933
.36	.56147	.83	.68402	2.20	.55968
.37	.56131	.84	.68652	2.30	.56007
.38	.56115	.85	.68656	2.40	.56045
.39	.56097	.86	.68351	2.50	.56078
.40	.56078	.87	.67662	3.00	.56182
.41	.56058	.88	.66511	3.50	.56223
.42	.56038	.89	.64814	4.00	.56239
.43	.56017	.90	.62493	4.50	.56245
.44	.55997	.91	.59483	5.00	.56248
.45	.55977	.92	.55749	5.50	.56249
.46	.55958	.93	.51309	6.00	.56250
.47	.55942	.94	.46262	6.50	.56250
.48	.55928	.95	.40818	7.00	.56250
.49	.55917	.96	.35318	7.50	.56250
.50	.55911	.97	.30224	8.00	.56250
.51	.55911	.98	.26067	8.50	.56250
.52	.55919	.99	.23348	9.00	.56250
.53	.55934	1.00	.22406	9.50	.56250
.54	.55960	1.01	.23329	10.00	.56250
.55	.55998	1.02	.25931		
.56	.56050	1.03	.29815		

APPENDIX E

<u>FORGO Programs for Aidentity Driving Point Impedance (ADPI) Calculations.</u> To make possible the computation of the ADPI versus frequency graph, the FORGO digital computer program shown on pages 80 and 81 was written and used. The input and output data for each of the previously discussed filter configurations are given on the pages following the computer program. The program would not handle Type 5, a fourteenth order polynomial, since it had a maximum capacity for a tenth order polynomial.

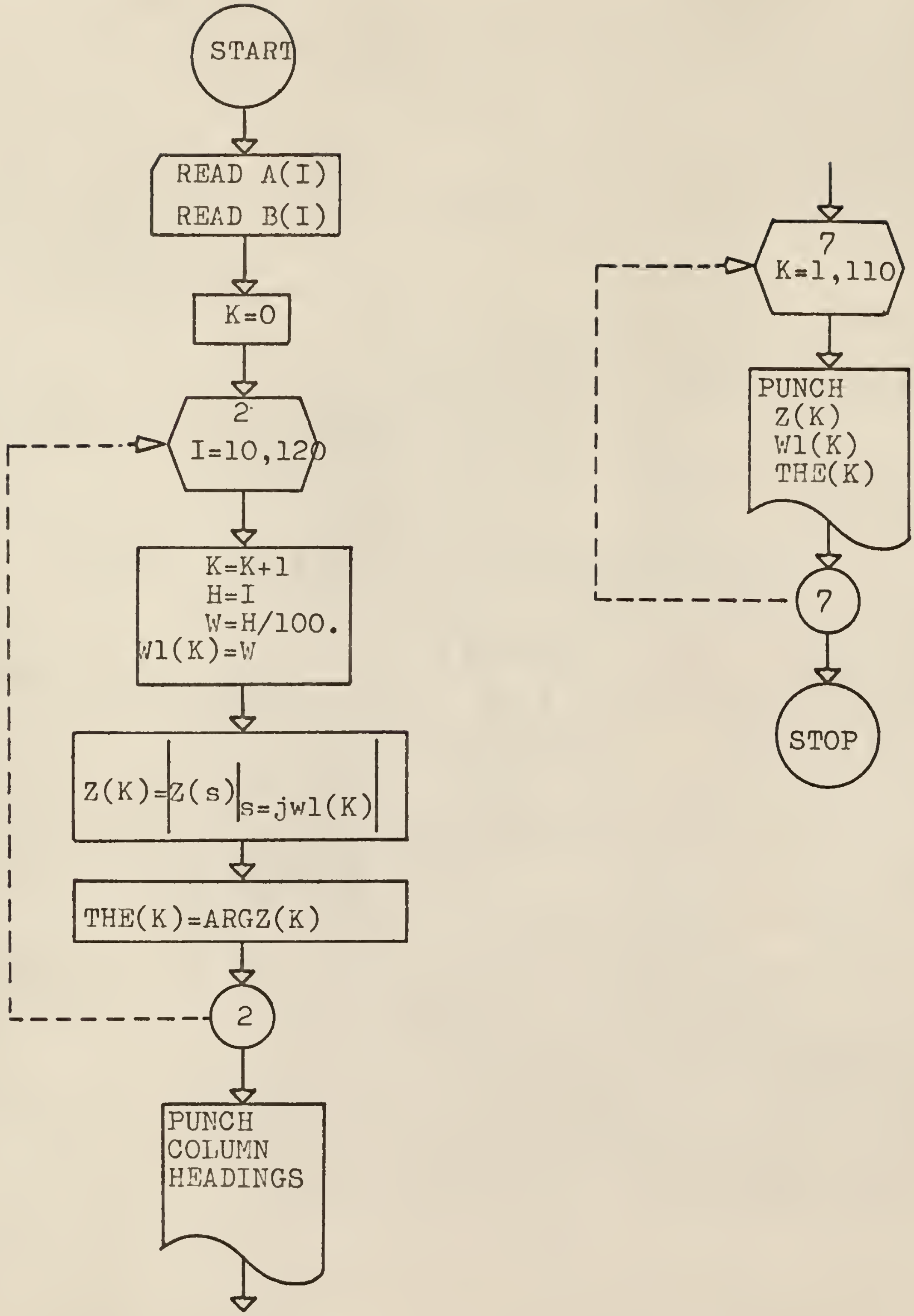

Fig. 55. Block diagram of ADPI computer program.

```
C   C   ADPI( Z(S) - PHASE SHIFT ) VS. FREQUENCY, ALL FILTERS

        DIMENSION A(10),B(10),W1(112),Z(112),THE(112)
      4 FORMAT(/7X,2HW1,5X,5HZ(JW),6X,2HO1,9X,2HW1,5X,5HZ(JW),6X,2HO1//)
      5 FORMAT( 2(5X,F5.2,3X,F7.5,2X,F7.2))
     30 FORMAT(7HW(K) = F5.2,5X,4HC = E15.8,5X,4HD = E15.8)
     31 FORMAT(7HW(K) = F5.2,5X,4HE = E15.8,5X,4HF = E15.8)
     45 READ,A0,(A(I),I=1,10),B0,(B(I),I=1,10)
        K=0
        DO 2 I=10,120
        K=K+1
        H=I
        W=H/100.
        W1(K)=W
        C=A0-A(2)*W**2+A(4)*W**4-A(6)*W**6+A(8)*W**8-A(10)*W**10
        D=A(1)*W-A(3)*W**3+A(5)*W**5-A(7)*W**7+A(9)*W**9
        E=B0-B(2)*W**2+B(4)*W**4-B(6)*W**6+B(8)*W**8-B(10)*W**10
        F=B(1)*W-B(3)*W**3+B(5)*W**5-B(7)*W**7+B(9)*W**9
        Z(K)=SQRT((C**2+D**2)/(E**2+F**2))
        IF(ABS(C)-.1E-40)3,3,8
      3 PUNCH 30,W1(K),C,D
        THE1=0.
        GO TO 14
      8 IF(C)10,10,11
     11 IF(D)12,12,13
     13 THE1=ATAN(D/C)*57.245779
        GO TO 14
     12 THE1=ATAN(D/C)*57.24779
        GO TO 14
     10 IF(D)15,15,16
     16 THE1=180.-ATAN(D/ABS(C))*57.245779
        GO TO 14
     15 THE1=180.+ATAN(D/C)*57.245779
     14 IF(ABS(E)-.1E-40)9,9,28
      9 PUNCH 31,W1(K),E,F
        THE2=0.
        GO TO 20
     28 IF(E)19,19,21
     21 IF(F)22,22,23
     23 THE2=ATAN(F/E)*57.245779
        GO TO 20
     22 THE2=ATAN(F/E)*57.245779
        GO TO 20
     19 IF(F)24,24,25
     25 THE2=180.-ATAN(F/ABS(E))*57.245779
        GO TO 20
     24 THE2=180.+ATAN(F/E)*57.245779
     20 THE(K)=THE1-THE2
      2 CONTINUE
        PUNCH 4
        DO 7 J=1,55
        PUNCH 5,W1(J),Z(J),THE(J),W1(J+55),Z(J+55),THE(J+55)
      7 CONTINUE
        GO TO 45
        END
```

```
ADPI RESPONSE, ADPI COEFFICIENTS - INPUT DATA, CONSTANT-1 FILTER
1.  2.  2.  2.  .0  .0  .0  .0  .0  .0  .0
1.  2.  2.  .0  .0  .0  .0  .0  .0  .0  .0


ADPI RESPONSE, ADPI COEFFICIENTS - INPUT DATA, ZOBEL FILTER, M=0.6
1.  3.84  5.76  11.63  6.32  9.1  1.95  2.07  .0  .0  .0
1.  3.2  8.06  6.64  10.81  3.07  3.26  .0  .0  .0  .0


ADPI RESPONSE, ADPI COEFFICIENTS - INPUT DATA, ZOBEL FILTER, M=0.707
1.  3.414  6.828  9.246  8.992  6.682  2.914  1.457  .0  .0  .0
1.  3.414  6.828  9.242  8.992  5.828  2.914  .0  .0  .0  .0


ADPI RESPONSE, ADPI COEFFICIENTS - INPUT DATA, TYPE 1 FILTER
1.  3.  5.  6.5  5.  3.  1.  .0  .0  .0  .0
1.  4.  8.  9.  8.  4.  1.  .0  .0  .0  .0


ADPI RESPONSE, ADPI COEFFICIENTS - INPUT DATA, TYPE 2 FILTER
3.24  11.7  21.55  28.79  21.55  11.7  3.24  .0  .0  .0  .0
3.24  11.7  22.8  25.42  22.8  11.7  3.24  .0  .0  .0  .0


ADPI RESPONSE, ADPI COEFFICIENTS - INPUT DATA, TYPE 3 FILTER
3.73246  13.92954  26.85862  36.3263  26.85862  13.92954  3.73246  .0
.0  .0  .0
3.73246  13.92954  26.8614  29.859  26.8614  13.92954  3.73246  .0  .0
.0  .0


ADPI RESPONSE, ADPI COEFFICIENTS - INPUT DATA, TYPE 4 FILTER
.9975  3.99  7.98  9.95  7.98  3.99  .9975  .0  .0  .0  .0
.9975  3.99  7.98  9.95  7.98  3.99  .9975  .0  .0  .0  .0


ADPI RESPONSE, ADPI COEFFICIENTS - INPUT DATA, TYPE 6 FILTER
3.23607 15.70820 43.12461 83.33938 119.37381 135.99185 119.37381
83.33938 43.12461 15.70820 3.23607        3.23607 15.70820 43.12461
81.48529 118.37383 131.8459 118.37383 81.48529 43.12461 15.70820 3.23607


ADPI RESPONSE, ADPI COEFFICIENTS - INPUT DATA, TYPE 7 FILTER
      .68140         3.63470         10.42514         20.45234 29.90444
    33.93411         29.904444       20.45234         10.42514 3.6347
      .68140
      .68140         3.63470         10.42514         20.45234 29.93704
    33.77306         29.93704        20.45234         10.42514 3.6347
      .6814(
```

C C ADPI(Z(S)-PHASE) VS. FREQUENCY, OUTPUT DATA, CONSTANT-1 FILTER, FIG.19

W1	Z(JW)	O1	W1	Z(JW)	O1
.10	.99960	-.11	.65	.58553	-4.86
.11	.99942	-.15	.66	.57007	-4.23
.13	.99887	-.24	.68	.53961	-2.71
.14	.99848	-.30	.69	.52473	-1.80
.15	.99800	-.37	.70	.51015	-.79
.16	.99742	-.44	.71	.49593	.34
.17	.99671	-.53	.72	.48212	1.57
.18	.99588	-.62	.73	.46879	2.93
.19	.99489	-.73	.74	.45601	4.41
.21	.99242	-.97	.76	.43232	7.75
.22	.99090	-1.10	.77	.42154	9.62
.23	.98916	-1.25	.78	.41157	11.62
.24	.98720	-1.40	.79	.40247	13.74
.25	.98498	-1.56	.80	.39429	15.98
.26	.98250	-1.74	.81	.38709	18.34
.27	.97974	-1.92	.82	.38093	20.80
.28	.97667	-2.12	.83	.37585	23.35
.29	.97329	-2.32	.84	.37188	25.96
.30	.96957	-2.53	.85	.36905	28.63
.31	.96549	-2.75	.86	.36736	31.34
.32	.96105	-2.98	.87	.36682	34.06
.33	.95622	-3.21	.88	.36742	36.76
.35	.94535	-3.70	.90	.37193	42.08
.36	.93928	-3.95	.91	.37575	44.64
.37	.93278	-4.20	.92	.38056	47.13
.38	.92582	-4.46	.93	.38630	49.53
.39	.91841	-4.71	.94	.39290	51.84
.40	.91054	-4.97	.95	.40031	54.03
.41	.90220	-5.22	.96	.40846	56.12
.42	.89339	-5.47	.97	.41728	58.10
.44	.87435	-5.95	.99	.43671	61.73
.45	.86412	-6.18	1.00	.44721	63.38
.46	.85344	-6.40	1.01	.45816	64.93
.47	.84229	-6.61	1.02	.46952	66.39
.48	.83070	-6.80	1.03	.48123	67.75
.49	.81867	-6.97	1.04	.49326	69.03
.50	.80623	-7.12	1.05	.50557	70.22
.51	.79338	-7.25	1.06	.51813	71.33
.52	.78014	-7.35	1.07	.53091	72.38
.53	.76654	-7.43	1.08	.54388	73.35
.54	.75260	-7.47	1.09	.55701	74.26
.55	.73835	-7.48	1.10	.57028	75.12
.56	.72381	-7.45	1.11	.58368	75.91
.58	.69398	-7.27	1.13	.61076	77.36
.59	.67876	-7.11	1.14	.62442	78.02
.60	.66339	-6.89	1.15	.63815	78.63
.61	.64789	-6.62	1.16	.65191	79.21
.62	.63232	-6.28	1.17	.66572	79.75
.63	.61671	-5.88	1.18	.67956	80.25
.64	.60109	-5.41	1.19	.69341	80.73

C C ADPI(Z(S)-PHASE) VS. FREQUENCY, OUTPUT DATA, ZOBEL FILTER, FIG.22

W1	Z(JW)	C1	W1	Z(JW)	C1
.10	1.04274	2.75	.65	.54949	10.78
.12	1.05972	2.84	.67	.56233	17.34
.13	1.06887	2.80	.68	.57453	20.59
.15	1.08798	2.54	.70	.61145	26.77
.16	1.09774	2.32	.71	.63658	29.63
.18	1.11705	1.68	.73	.70144	34.75
.19	1.12634	1.25	.74	.74188	36.97
.20	1.13519	.77	.75	.78831	38.93
.21	1.14346	.21	.76	.84138	40.64
.22	1.15098	-.40	.77	.90198	42.07
.23	1.15761	-1.08	.78	.97125	43.23
.25	1.16755	-2.60	.80	1.14217	44.63
.26	1.17055	-3.44	.81	1.24835	44.82
.27	1.17206	-4.32	.82	1.37262	44.61
.29	1.17011	-6.20	.84	1.69520	42.68
.30	1.16644	-7.18	.85	1.90766	40.71
.31	1.16089	-8.18	.86	2.16725	37.80
.32	1.15341	-9.18	.87	2.48585	33.60
.33	1.14398	-10.18	.88	2.87282	27.63
.34	1.13262	-11.18	.89	3.32127	19.23
.35	1.11938	-12.16	.90	3.77532	7.77
.36	1.10430	-13.11	.91	4.09281	-6.76
.37	1.08749	-14.02	.92	4.10160	-22.88
.38	1.06905	-14.88	.93	3.78655	321.88
.39	1.04911	-15.69	.94	3.31218	309.64
.40	1.02783	-16.59	.95	2.83484	300.43
.41	1.00535	-17.27	.96	2.42098	293.66
.42	.98185	-17.85	.97	2.08054	288.67
.43	.95748	-18.35	.98	1.80385	284.92
.44	.93244	-18.75	.99	1.57788	282.05
.45	.90688	-19.04	1.00	1.39124	279.80
.46	.88098	-19.21	1.01	1.23501	278.02
.47	.85491	-19.25	1.02	1.10248	276.58
.48	.82884	-19.00	1.03	.98861	275.40
.49	.80292	-18.77	1.04	.88962	274.42
.50	.77731	-18.38	1.05	.80264	273.60
.51	.75218	-17.83	1.06	.72546	272.89
.52	.72769	-17.10	1.07	.65638	272.28
.53	.70398	-16.19	1.08	.59406	271.75
.54	.68123	-15.09	1.09	.53743	271.28
.55	.65961	-13.78	1.10	.48562	270.86
.56	.63929	-12.26	1.11	.43796	270.48
.57	.62046	-10.53	1.12	.39386	270.12
.58	.60333	-8.56	1.13	.35286	269.79
.59	.58809	-6.38	1.14	.31457	269.47
.60	.57498	-3.97	1.15	.27866	269.15
.61	.56422	-1.36	1.16	.24486	268.82
.62	.55605	1.46	1.17	.21292	268.49
.63	.55072	4.44	1.18	.18265	268.12
.64	.54845	7.56	1.19	.15388	267.69

C C ADPI(Z(S)-PHASE) VS. FREQUENCY, OUTPUT DATA, ZOBEL FILTER, FIG.25

W1	Z(JW)	O1	W1	Z(JW)	O1
.10	1.00000	.00	.65	1.02577	-1.96
.12	1.00001	.00	.67	1.02217	-2.11
.13	1.00001	.00	.68	1.01987	-2.14
.15	1.00002	.00	.70	1.01446	-2.07
.17	1.00005	.00	.72	1.00851	-1.77
.19	1.00011	.00	.74	1.00312	-1.16
.20	1.00015	.01	.75	1.00116	-.72
.22	1.00026	.02	.77	1.00028	.52
.23	1.00034	.02	.78	1.00221	1.33
.24	1.00044	.02	.79	1.00644	2.27
.25	1.00055	.03	.80	1.01368	3.34
.26	1.00070	.03	.81	1.02477	4.56
.27	1.00086	.04	.82	1.04076	5.89
.28	1.00106	.04	.83	1.06290	7.34
.29	1.00130	.05	.84	1.09277	8.86
.30	1.00157	.05	.85	1.13232	10.40
.31	1.00188	.06	.86	1.18399	11.92
.32	1.00224	.06	.87	1.25089	13.32
.33	1.00264	.07	.88	1.33705	14.50
.34	1.00310	.07	.89	1.44774	15.32
.35	1.00362	.07	.90	1.58996	15.58
.36	1.00420	.07	.91	1.77286	15.03
.37	1.00484	.07	.92	2.00788	13.34
.38	1.00554	.07	.93	2.30713	9.99
.39	1.00631	.07	.94	2.67559	4.33
.40	1.00716	.06	.95	3.08795	-4.45
.41	1.00807	.05	.96	3.44631	-16.76
.42	1.00905	.04	.97	3.58132	-31.62
.43	1.01010	.02	.98	3.40664	-46.37
.44	1.01121	-.00	.99	3.03443	-58.57
.45	1.01238	-.03	1.00	2.62587	-67.56
.46	1.01361	-.07	1.01	2.26314	-73.89
.47	1.01489	-.11	1.02	1.96429	-78.30
.48	1.01621	-.16	1.03	1.72282	-81.42
.49	1.01755	-.21	1.04	1.52714	-83.64
.50	1.01891	-.28	1.05	1.36673	-85.24
.51	1.02027	-.35	1.06	1.23332	-86.43
.52	1.02161	-.43	1.07	1.12073	-87.30
.53	1.02291	-.52	1.08	1.02435	-87.96
.54	1.02415	-.61	1.09	.94078	-88.46
.55	1.02530	-.72	1.10	.86745	-88.84
.56	1.02634	-.83	1.11	.80243	-89.13
.57	1.02723	-.95	1.12	.74422	-89.36
.58	1.02796	-1.08	1.13	.69165	-89.54
.59	1.02848	-1.21	1.14	.64381	-89.67
.60	1.02877	-1.34	1.15	.59996	-89.78
.61	1.02880	-1.48	1.16	.55954	-89.87
.62	1.02854	-1.61	1.17	.52205	-89.93
.63	1.02796	-1.74	1.18	.48711	-89.99
.64	1.02704	-1.86	1.19	.45439	-90.03

C C ADPI(Z(S)-PHASE) VS. FREQUENCY, OUTPUT DATA, TYPE 1 FILTER, FIG.28

W1	Z(JW)	C1	W1	Z(JW)	C1
.10	.99440	-5.84	.65	.49916	-30.07
.12	.99157	-7.07	.67	.47600	-28.91
.15	.98581	-8.96	.70	.44404	-26.53
.18	.97785	-10.92	.73	.41636	-23.32
.19	.97463	-11.59	.74	.40831	-22.05
.20	.97111	-12.27	.75	.40093	-20.69
.21	.96726	-12.95	.76	.39430	-19.24
.22	.96307	-13.64	.77	.38847	-17.69
.23	.95853	-14.33	.78	.38351	-16.06
.24	.95362	-15.03	.79	.37950	-14.36
.25	.94834	-15.73	.80	.37651	-12.59
.26	.94266	-16.43	.81	.37462	-10.77
.27	.93659	-17.14	.82	.37389	-8.93
.28	.93012	-17.85	.83	.37440	-7.09
.29	.92323	-18.55	.84	.37620	-5.26
.30	.91594	-19.26	.85	.37933	-3.49
.31	.90822	-19.96	.86	.38383	-1.81
.32	.90010	-20.66	.87	.38968	-.24
.33	.89156	-21.35	.88	.39687	1.18
.34	.88262	-22.04	.89	.40530	2.41
.35	.87328	-22.72	.90	.41487	3.43
.36	.86355	-23.38	.91	.42537	4.21
.37	.85344	-24.04	.92	.43655	4.71
.38	.84297	-24.68	.93	.44808	4.95
.39	.83215	-25.46	.94	.45953	4.90
.40	.82099	-26.07	.95	.47044	4.57
.41	.80952	-26.66	.96	.48029	3.99
.42	.79776	-27.23	.97	.48857	3.20
.43	.78571	-27.78	.98	.49482	2.23
.44	.77342	-28.31	.99	.49869	1.14
.45	.76089	-28.80	1.00	.50000	0.00
.46	.74816	-29.27	1.01	.49872	-1.13
.47	.73524	-29.72	1.02	.49502	-2.19
.48	.72216	-30.12	1.03	.48920	-3.12
.49	.70895	-30.50	1.04	.48168	-3.89
.50	.69563	-30.84	1.05	.47290	-4.46
.51	.68222	-31.14	1.06	.46332	-4.82
.52	.66875	-31.25	1.07	.45336	-4.96
.53	.65524	-31.47	1.08	.44337	-4.89
.54	.64172	-31.65	1.09	.43363	-4.61
.55	.62821	-31.79	1.10	.42439	-4.15
.56	.61474	-31.87	1.11	.41578	-3.51
.57	.60133	-31.91	1.12	.40793	-2.73
.58	.58800	-31.89	1.13	.40090	-1.82
.59	.57478	-31.82	1.14	.39472	-.80
.60	.56170	-31.69	1.15	.38940	.30
.61	.54877	-31.50	1.16	.38493	1.47
.62	.53603	-31.25	1.17	.38127	2.69
.63	.52349	-30.92	1.18	.37841	3.94
.64	.51119	-30.53	1.19	.37628	5.20

C C ADPI(Z(S)-PHASE) VS. FREQUENCY, OUTPUT DATA, TYPE 2 FILTER, FIG.31

W1	Z(JW)	O1	W1	Z(JW)	O1
.10	1.00322	-.13	.65	.76985	12.63
.13	1.00471	-.28	.68	.78165	17.25
.15	1.00550	-.43	.70	.79891	20.48
.17	1.00595	-.60	.72	.82508	23.71
.19	1.00589	-.81	.74	.86144	26.85
.20	1.00562	-.93	.75	.88389	28.34
.21	1.00516	-1.06	.76	.90944	29.77
.22	1.00448	-1.19	.77	.93830	31.12
.23	1.00357	-1.33	.78	.97070	32.37
.24	1.00241	-1.48	.79	1.00690	33.52
.25	1.00097	-1.63	.80	1.04717	34.54
.26	.99923	-1.78	.81	1.09181	35.43
.27	.99718	-1.94	.82	1.14118	36.16
.28	.99480	-2.09	.83	1.19562	36.73
.29	.99206	-2.25	.84	1.25554	37.11
.30	.98897	-2.41	.85	1.32136	37.29
.31	.98550	-2.56	.86	1.39350	37.25
.32	.98164	-2.71	.87	1.47237	36.96
.33	.97739	-2.85	.88	1.55832	36.40
.34	.97274	-2.98	.89	1.65157	35.54
.35	.96769	-3.10	.90	1.75214	34.34
.36	.96225	-3.21	.91	1.85966	32.78
.37	.95640	-3.30	.92	1.97323	30.82
.38	.95017	-3.37	.93	2.09113	28.43
.39	.94357	-3.42	.94	2.21061	25.58
.40	.93660	-3.44	.95	2.32763	22.25
.41	.92929	-3.44	.96	2.43681	18.47
.42	.92165	-3.56	.97	2.53171	14.26
.43	.91372	-3.34	.98	2.60555	9.70
.44	.90552	-3.23	.99	2.65236	4.90
.45	.89710	-3.09	1.00	2.66832	0.00
.46	.88847	-2.90	1.01	2.65267	-4.85
.47	.87970	-2.66	1.02	2.60793	-9.52
.48	.87082	-2.38	1.03	2.53910	-13.88
.49	.86189	-2.04	1.04	2.45251	-17.85
.50	.85295	-1.64	1.05	2.35456	-21.40
.51	.84407	-1.19	1.06	2.25087	-24.50
.52	.83532	-.67	1.07	2.14586	-27.18
.53	.82675	-.08	1.08	2.04272	-29.46
.54	.81844	.57	1.09	1.94355	-31.37
.55	.81048	1.30	1.10	1.84962	-32.94
.56	.80293	2.09	1.11	1.76155	-34.22
.57	.79589	2.97	1.12	1.67957	-35.23
.58	.78945	3.92	1.13	1.60361	-36.01
.59	.78370	4.94	1.14	1.53346	-36.59
.60	.77875	6.05	1.15	1.46879	-36.98
.61	.77470	7.22	1.16	1.40925	-37.21
.62	.77167	8.47	1.17	1.35446	-37.30
.63	.76977	9.80	1.18	1.30404	-37.27
.64	.76912	11.18	1.19	1.25766	-37.12

C C ADPI(Z(S)-PHASE) VS. FREQUENCY, OUTPUT DATA, TYPE 3 FILTER, FIG.34

W1	Z(JW)	O1	W1	Z(JW)	O1
.10	.99937	-.09	.65	.80153	25.89
.13	.99823	-.19	.68	.84372	31.14
.15	.99689	-.29	.70	.88441	34.53
.17	.99493	-.39	.72	.93662	37.73
.19	.99220	-.52	.74	1.00188	40.65
.20	.99051	-.59	.75	1.03995	41.98
.21	.98857	-.65	.76	1.08202	43.21
.22	.98637	-.72	.77	1.12839	44.34
.23	.98389	-.79	.78	1.17943	45.36
.24	.98111	-.86	.79	1.23555	46.26
.25	.97802	-.92	.80	1.29724	47.03
.26	.97461	-.99	.81	1.36506	47.65
.27	.97087	-1.04	.82	1.43964	48.13
.28	.96679	-1.09	.83	1.52174	48.45
.29	.96235	-1.12	.84	1.61222	48.59
.30	.95756	-1.15	.85	1.71205	48.54
.31	.95241	-1.16	.86	1.82236	48.28
.32	.94691	-1.16	.87	1.94438	47.77
.33	.94106	-1.13	.88	2.07945	46.99
.34	.93486	-1.09	.89	2.22898	45.91
.35	.92832	-1.02	.90	2.39429	44.47
.36	.92147	-.92	.91	2.57645	42.63
.37	.91432	-.79	.92	2.77585	40.33
.38	.90688	-.63	.93	2.99170	37.49
.39	.89919	-.44	.94	3.22103	34.05
.40	.89127	-.20	.95	3.45764	29.94
.41	.88316	.08	.96	3.69080	25.12
.42	.87490	.40	.97	3.90464	19.60
.43	.86653	.78	.98	4.07917	13.45
.44	.85810	1.21	.99	4.19390	6.83
.45	.84966	1.69	1.00	4.23378	0.00
.46	.84127	2.24	1.01	4.19468	-6.77
.47	.83298	2.85	1.02	4.08492	-13.20
.48	.82487	3.52	1.03	3.92177	-19.09
.49	.81700	4.27	1.04	3.72542	-24.32
.50	.80946	5.09	1.05	3.51397	-28.86
.51	.80232	5.98	1.06	3.30100	-32.73
.52	.79566	6.95	1.07	3.09533	-35.99
.53	.78959	7.99	1.08	2.90190	-38.72
.54	.78420	9.12	1.09	2.72300	-40.97
.55	.77959	10.32	1.10	2.55917	-42.82
.56	.77586	11.60	1.11	2.41000	-44.33
.57	.77313	12.95	1.12	2.27455	-45.54
.58	.77150	14.38	1.13	2.15166	-46.50
.59	.77111	15.87	1.14	2.04013	-47.24
.60	.77206	17.43	1.15	1.93881	-47.80
.61	.77449	19.04	1.16	1.84660	-48.19
.62	.77852	20.70	1.17	1.76253	-48.45
.63	.78428	22.40	1.18	1.68572	-48.58
.64	.79191	24.14	1.19	1.61542	-48.60

C C ADPI(Z(S)-PHASE) VS. FREQUENCY, OUTPUT DATA, TYPE 6 FILTER, FIG.42

W1	Z(JW)	O1	W1	Z(JW)	O1
.10	.99976	-.03	.65	.96903	5.64
.12	.99951	-.05	.67	.97437	6.04
.14	.99912	-.07	.69	.97964	6.40
.15	.99885	-.09	.70	.98210	6.57
.17	.99814	-.12	.72	.98634	6.90
.19	.99718	-.15	.74	.98922	7.25
.20	.99659	-.17	.75	.99001	7.45
.22	.99517	-.20	.77	.99013	7.96
.23	.99434	-.21	.78	.98949	8.29
.24	.99341	-.22	.79	.98846	-351.15
.25	.99240	-.23	.80	.98719	-350.66
.26	.99129	-.24	.81	.98588	9.79
.27	.99009	-.24	.82	.98486	10.53
.28	.98881	-.24	.83	.98456	11.42
.29	.98743	-.23	.84	.98559	12.48
.30	.98597	-.22	.85	.98871	13.74
.31	.98443	-.20	.86	.99494	15.20
.32	.98281	-.18	.87	1.00555	16.86
.33	.98112	-.14	.88	1.02216	18.72
.34	.97937	-.10	.89	1.04679	20.73
.35	.97757	-.05	.90	1.08194	22.83
.36	.97572	.00	.91	1.13074	24.93
.37	.97384	.07	.92	1.19704	26.87
.38	.97195	.15	.93	1.28561	28.47
.39	.97004	.24	.94	1.40212	29.44
.40	.96815	.34	.95	1.55284	29.46
.41	.96629	.45	.96	1.74272	28.06
.42	.96446	.58	.97	1.97021	24.70
.43	.96269	.72	.98	2.21421	18.83
.44	.96100	.86	.99	2.41842	10.32
.45	.95940	1.02	1.00	2.50065	0.00
.46	.95792	1.20	1.01	2.41999	-10.22
.47	.95656	1.38	1.02	2.22346	-18.55
.48	.95536	1.58	1.03	1.99142	-24.29
.49	.95432	1.78	1.04	1.77552	-27.69
.50	.95347	2.00	1.05	1.59440	-29.27
.51	.95282	2.23	1.06	1.44918	-29.58
.52	.95239	2.46	1.07	1.33512	-29.01
.53	.95219	2.70	1.08	1.24649	-27.87
.54	.95223	2.95	1.09	1.17809	-26.40
.55	.95253	3.20	1.10	1.12564	-24.74
.56	.95308	3.45	1.11	1.08572	-23.03
.57	.95390	3.71	1.12	1.05563	-21.33
.58	.95498	3.97	1.13	1.03321	-19.70
.59	.95632	4.22	1.14	1.01679	-18.18
.60	.95791	4.48	1.15	1.00497	-16.79
.61	.95975	4.72	1.16	.99673	-15.53
.62	.96180	4.96	1.17	.99118	-14.40
.63	.96406	5.20	1.18	.98767	-13.40
.64	.96648	5.42	1.19	.98566	-12.53

C C ADPI(Z(S)-PHASE) VS. FREQUENCY, OUTPUT DATA, TYPE 7 FILTER, FIG.45

W1	Z(JW)	C1	W1	Z(JW)	C1
.10	1.00000	.00	.65	.97784	-2.36
.12	1.00000	.00	.67	.96597	-2.56
.14	1.00000	.00	.69	.95132	-2.65
.15	1.00000	.00	.70	.94294	-2.63
.17	1.00000	.00	.72	.92414	-2.43
.19	1.00002	.00	.74	.90300	-1.92
.20	1.00003	.00	.75	.89179	-1.51
.22	1.00008	.00	.77	.86886	-.33
.23	1.00011	.01	.78	.85759	.48
.24	1.00015	.01	.79	.84682	1.47
.25	1.00020	.01	.80	.83693	2.64
.26	1.00026	.02	.81	.82836	4.01
.27	1.00034	.02	.82	.82165	5.60
.28	1.00043	.02	.83	.81744	7.41
.29	1.00053	.02	.84	.81648	9.44
.30	1.00066	.02	.85	.81967	11.68
.31	1.00080	.02	.86	.82806	14.09
.32	1.00096	.02	.87	.84285	16.64
.33	1.00115	.02	.88	.86548	19.27
.34	1.00135	.02	.89	.89762	21.88
.35	1.00158	.01	.90	.94120	24.36
.36	1.00184	.00	.91	.99854	26.59
.37	1.00211	-.00	.92	1.07233	28.40
.38	1.00241	-.01	.93	1.16570	29.61
.39	1.00273	-.03	.94	1.28203	30.00
.40	1.00307	-.04	.95	1.42418	29.27
.41	1.00342	-.06	.96	1.59257	27.12
.42	1.00379	-.08	.97	1.78068	23.17
.43	1.00415	-.11	.98	1.96766	17.16
.44	1.00452	-.15	.99	2.11307	9.19
.45	1.00487	-.18	1.00	2.16889	90.08
.46	1.00521	-.23	1.01	2.11415	-9.11
.47	1.00551	-.28	1.02	1.97447	-16.88
.48	1.00576	-.34	1.03	1.79753	-22.72
.49	1.00595	-.40	1.04	1.62057	-26.63
.50	1.00606	-.48	1.05	1.46201	-28.90
.51	1.00607	-.56	1.06	1.32734	-29.89
.53	1.00568	-.75	1.08	1.12512	-29.20
.54	1.00523	-.85	1.09	1.05166	-27.98
.55	1.00457	-.97	1.10	.99271	-26.40
.56	1.00366	-1.09	1.11	.94579	-24.57
.57	1.00247	-1.22	1.12	.90883	-22.60
.58	1.00096	-1.36	1.13	.88012	-20.57
.59	.99908	-1.50	1.14	.85826	-18.53
.60	.99681	-1.65	1.15	.84207	-16.53
.61	.99408	-1.80	1.16	.83056	-14.61
.62	.99086	-1.95	1.17	.82291	-12.79
.63	.98711	-2.09	1.18	.81842	-11.09
.64	.98278	-2.23	1.19	.81652	-9.52

APPENDIX F

Voltage Transfer Function (VTF) Coefficients. To determine the VTF frequency response of the previously discussed filters, it is necessary to know the VTF coefficients. Since all of the filters, with the exception of the constant-1 filter, have the same general ladder configuration of Fig. 56, a pseudo-generalized computer program was written to determine these coefficients. First the transfer matrix of this

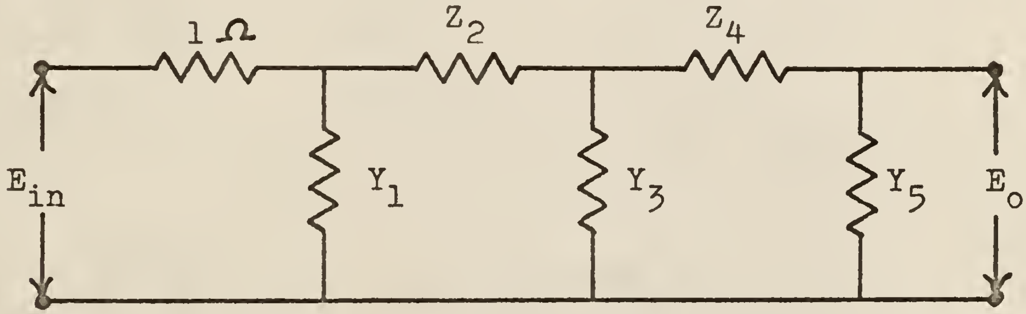

Fig. 56. Generalized ladder configuration of filters being analyzed.

generalized ladder network was determined and the inverse of the (1,1) element was used as the VTF, α. The generalized expression for α is given in equation (44). Equations

$$\alpha = 1+Y_1+Y_3+Y_5+Z_2Y_3+Z_2Y_5+Z_4Y_5+Y_1Z_2Y_3+Y_1Z_4Y_5$$
$$+Y_1Z_2Y_5+Y_3Z_4Y_5+Z_2Y_3Z_4Y_5+Y_1Z_2Y_3Z_4Y_5 \qquad (44)$$

$$Y_n = \frac{Y_{nN}}{Y_{nD}} \text{ and } Z_n = \frac{Z_{nN}}{Z_{nD}} \text{ and } \alpha = \frac{\alpha_N}{\alpha_D}$$

$$\alpha_N = Y_{1D}Z_{2D}Y_{3D}Z_{4D}Y_{5D} \qquad (45)$$

$$\mathcal{L}_D = \sum_{n=1}^{13} B_n \tag{46}$$

where

$$
\begin{aligned}
B_1 &= Y_{1D}Z_{2D}Y_{3D}Z_{4D}Y_{5D} & B_7 &= Z_{4N}Y_{5N}Y_{1D}Z_{2D}Y_{3D} \\
B_2 &= Y_{1N}Z_{2D}Y_{3D}Z_{4D}Y_{5D} & B_8 &= Y_{1N}Z_{2N}Y_{3N}Z_{4D}Y_{5D} \\
B_3 &= Y_{3N}Y_{1D}Z_{2D}Z_{4D}Y_{5D} & B_9 &= Y_{1N}Z_{4N}Y_{5N}Z_{2D}Y_{3D} \\
B_4 &= Y_{5N}Y_{1D}Z_{2D}Y_{3D}Z_{4D} & B_{10} &= Y_{1N}Z_{2N}Y_{5N}Y_{3D}Z_{4D} \\
B_5 &= Z_{2N}Y_{3N}Y_{1D}Z_{4D}Y_{5D} & B_{11} &= Y_{3N}Z_{4N}Y_{5N}Y_{1D}Z_{2D} \\
B_6 &= Z_{2N}Y_{5N}Y_{1D}Y_{3D}Z_{4D} & B_{12} &= Z_{2N}Y_{3N}Z_{4N}Y_{5N}Y_{1D} \\
& & B_{13} &= Y_{1N}Z_{2N}Y_{3N}Z_{4N}Y_{5N}
\end{aligned} \tag{47}
$$

(45), (46), and (47) put $\mathcal{L}$ in the form which the computer program can handle it. The following restrictions were placed on ladder component expressions to simplify the computer program:

1. $Z_{2D} = Y_{3D} = Z_{4D} = 1$ for all types.
2. Z_{2N}, Y_{3N}, and Z_{4N} are of the form - As.
3. Y_{1N} and Y_{1D} are a maximum of fifth order polynomials.
4. Y_{5N} and Y_{5D} are a maximum of second order polynomials.

Using these restrictions, $\mathcal{L}_D$ reduces to equation (48) and $\mathcal{L}_N$ reduces to equation (49). With these restrictions, Type 5 exceeds the restrictions on Y_{1N} and Y_{1D} so it was ignored.

$$\mathcal{L}_D = Y_{1D}Y_{5D}(1+Y_{3N}+Z_2Y_{3N}) + Y_{1N}Y_{5D}(1+Z_{2N}Y_{3N}) + Y_{1D}Y_{5N}(1+Z_{2N}+Z_{4N}+Y_{3N}Z_{4N}+Z_{2N}Y_{3N}Z_{4N}) + Y_{1N}Y_{5N}(Z_{2N}+Z_{4N}+Z_{2N}Y_{3N}Z_{4N}) \tag{48}$$

$$\mathcal{L}_N = Y_{1D}Y_{5D} \tag{49}$$

APPENDIX G

FORGO Programs for VTF Coefficient Calculations. The computer program on pages 94 through 97 was written to evaluate the coefficients of equations (45), (48) and (49). The input and output data for all filters except the constant-1 filter and Type 5 are given on the pages following the computer program.

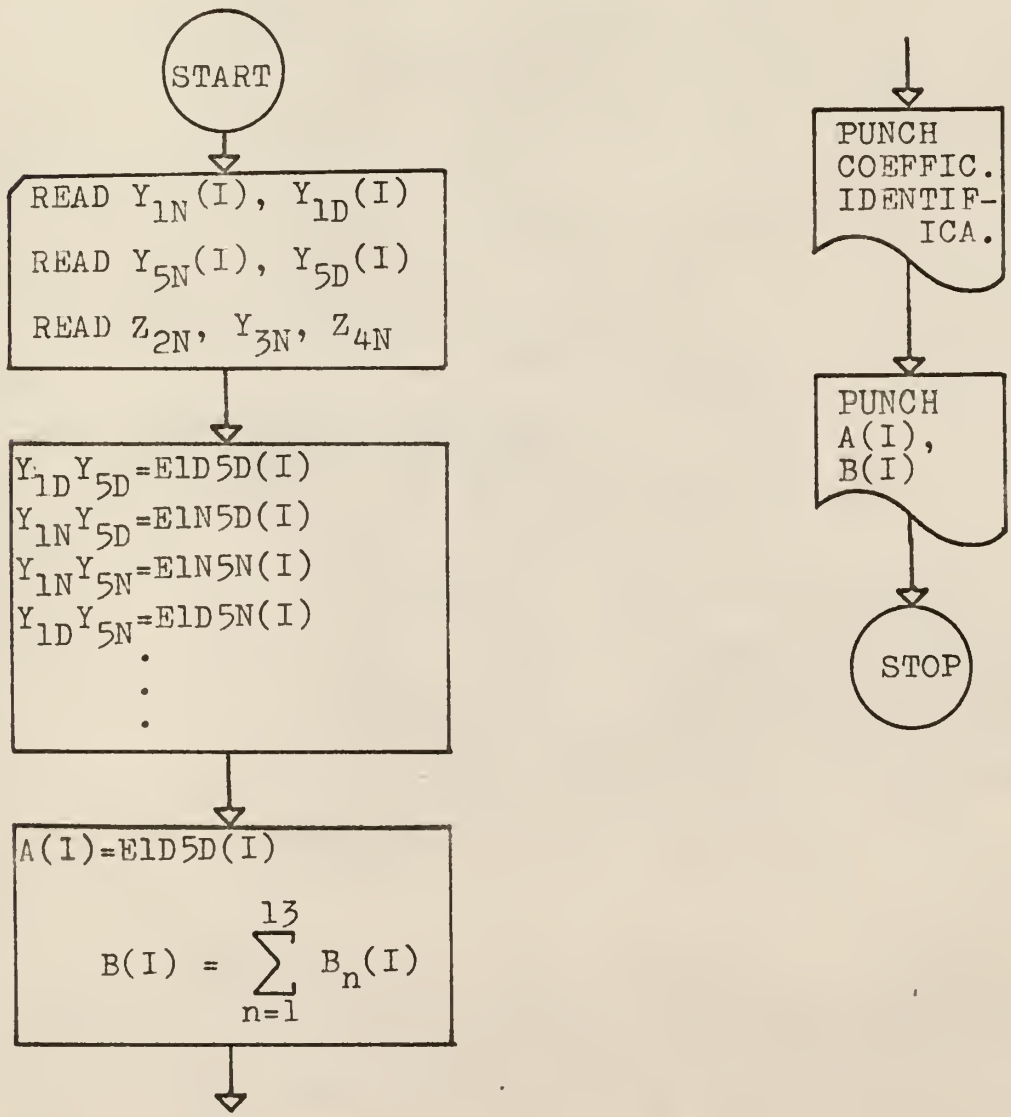

Fig. 57. Block diagram of VTF coefficient computer program.

```
C  C  VOLTAGE TRANSFER FUNCTION(VTF) COEFFICIENT EVALUATION
      DIMENSION E1N5D(10),E1N5N(10),E1D5N(10),E1D5D(10),A1(10),B1(10)
      DIMENSION B2(10),B3(10),B4(10),B5(10),B6(10),B7(10),B8(10),B9(10)
      DIMENSION B10(10),B11(10),B12(10),B13(10),B(10),Y1N(5),Y1D(5)
      DIMENSION Y5N(3),Y5D(3)
   18 READ,Y1N0,(Y1N(I),I=1,5),Y1D0,(Y1D(I),I=1,5)
      READ,Z2N,Y3N,Z4N
      READ,Y5N0,Y5N(1),Y5N(2),Y5D0,Y5D(1),Y5D(2)
      E1ND0 =Y1N0*Y5D0
      E1N5D(1)=Y1N(1)*Y5D0+Y1N0*Y5D(1)
      E1N5D(2)=Y1N(2)*Y5D0+Y1N(1)*Y5D(1)+Y1N0*Y5D(2)
      E1N5D(3)=Y1N(3)*Y5D0+Y1N(2)*Y5D(1)+Y1N(1)*Y5D(2)
      E1N5D(4)=Y1N(4)*Y5D0+Y1N(3)*Y5D(1)+Y1N(2)*Y5D(2)
      E1N5D(5)=Y1N(5)*Y5D0+Y1N(4)*Y5D(1)+Y1N(3)*Y5D(2)
      E1N5D(6)=            Y1N(5)*Y5D(1)+Y1N(4)*Y5D(2)
      E1N5D(7)=                          Y1N(5)*Y5D(2)
      E1NN0 =Y1N0*Y5N0
      E1N5N(1)=Y1N(1)*Y5N0+Y1N0*Y5N(1)
      E1N5N(2)=Y1N(2)*Y5N0+Y1N(1)*Y5N(1)+Y1N0*Y5N(2)
      E1N5N(3)=Y1N(3)*Y5N0+Y1N(2)*Y5N(1)+Y1N(1)*Y5N(2)
      E1N5N(4)=Y1N(4)*Y5N0+Y1N(3)*Y5N(1)+Y1N(2)*Y5N(2)
      E1N5N(5)=Y1N(5)*Y5N0+Y1N(4)*Y5N(1)+Y1N(3)*Y5N(2)
      E1N5N(6)=            Y1N(5)*Y5N(1)+Y1N(4)*Y5N(2)
      E1N5N(7)=                          Y1N(5)*Y5N(2)
      E1DN0 =Y1D0*Y5N0
      E1D5N(1)=Y1D(1)*Y5N0+Y1D0*Y5N(1)
      E1D5N(2)=Y1D(2)*Y5N0+Y1D(1)*Y5N(1)+Y1D0*Y5N(2)
      E1D5N(3)=Y1D(3)*Y5N0+Y1D(2)*Y5N(1)+Y1D(1)*Y5N(2)
      E1D5N(4)=Y1D(4)*Y5N0+Y1D(3)*Y5N(1)+Y1D(2)*Y5N(2)
      E1D5N(5)=Y1D(5)*Y5N0+Y1D(4)*Y5N(1)+Y1D(3)*Y5N(2)
      E1D5N(6)=            Y1D(5)*Y5N(1)+Y1D(4)*Y5N(2)
      E1D5N(7)=                          Y1D(5)*Y5N(2)
      E1DD0 =Y1D0*Y5D0
      E1D5D(1)=Y1D(1)*Y5D0+Y1D0*Y5D(1)
      E1D5D(2)=Y1D(2)*Y5D0+Y1D(1)*Y5D(1)+Y1D0*Y5D(2)
      E1D5D(3)=Y1D(3)*Y5D0+Y1D(2)*Y5D(1)+Y1D(1)*Y5D(2)
      E1D5D(4)=Y1D(4)*Y5D0+Y1D(3)*Y5D(1)+Y1D(2)*Y5D(2)
      E1D5D(5)=Y1D(5)*Y5D0+Y1D(4)*Y5D(1)+Y1D(3)*Y5D(2)
      E1D5D(6)=            Y1D(5)*Y5D(1)+Y1D(4)*Y5D(2)
      E1D5D(7)=                          Y1D(5)*Y5D(2)
      F=Y3N
      F1=Z2N*Y3N
      G=Z2N
      P=Z4N
      F2=Y3N*Z4N
      F3=F2*G
      A10=E1DD0
      DO 1 I=1,7
    1 A1(I)=E1D5D(I)
      A1(8)=0.0
      A1(9)=0.0
      A1(10)=0.0
      B10Z=A10
      DO 2 I=1,10
    2 B1(I)=A1(I)
```

```
      B20=E1ND0
      DO 3 I=1,7
    3 B2(I)=E1N5D(I)
      B2(8)=0.0
      B2(9)=0.0
      B2(10)=0.0
      B30=0.0
      B3(1)=F*E1DD0
      DO 4 I=2,8
    4 B3(I)=F*E1D5D(I-1)
      B3(9)=0.0
      B3(10)=0.0
      B40=E1DN0
      DO 5 I=1,7
    5 B4(I)=E1D5N(I)
      B4(8)=0.0
      B4(9)=0.0
      B4(10)=0.0
      B50=0.0
      B5(1)=0.0
      B5(2)=F1*E1DD0
      DO 6 I=3,9
    6 B5(I)=F1*E1D5D(I-2)
      B5(10)=0.0
      B60=0.0
      B6(1)=G*E1DN0
      DO 7 I=2,8
    7 B6(I)=G*E1D5N(I-1)
      B6(9)=0.0
      B6(10)=0.0
      B70=0.0
      B7(1)=P*E1DN0
      DO 8 I=2,8
    8 B7(I)=P*E1D5N(I-1)
      B7(9)=0.0
      B7(10)=0.0
      B80=0.0
      B8(1)=0.0
      B8(2)=F1*E1ND0
      DO 9 I=3,9
    9 B8(I)=F1*E1N5D(I-2)
      B8(10)=0.0
      B90=0.0
      B9(1)=P*E1NN0
      DO 10 I=2,8
   10 B9(I)=P*E1N5N(I-1)
      B9(9)=0.0
      B9(10)=0.0
      B100=0.0
      B10(1)=G*E1NN0
      DO 11 I=2,8
   11 B10(I)=G*E1N5N(I-1)
      B10(9)=0.0
      B10(10)=0.0
      B110=0.0
```

```
      B11(1)=0.0
      B11(2)=F2*E1DN0
      DO 12 I=3,9
   12 B11(I)=F2*E1D5N(I-2)
      B11(10)=).0
      B120=0.0
      B12(1)=0.0
      B12(2)=0.0
      B12(3)=F3*E1DN0
      DO 13 I=4,10
   13 B12(I)=F3*E1D5N(I-3)
      B130=0.0
      B13(1)=0.0
      B13(2)=0.0
      B13(3)=F3*E1NN0
      DO 14 I=4,10
   14 B13(I)=F3*E1N5N(I-3)
      B0=B10Z+B20+B40
      DO 19 I=1,10
      BT = B1(I)+B2(I)+B3(I)+B4(I)+B5(I)+B6(I)+B7(I)+B8(I)+B9(I)+B10(I)
   19 B(I)=BT +B11(I)+B12(I)+B13(I)
      PUNCH 15,A10,B0
   15 FORMAT(5X,10H A( 0)  = F10.5,5X,10H B( 0)  = F10.5)
      DO 16 I=1,10
   16 PUNCH 17,I,A1(I),I,B(I)
   17 FORMAT(5X,3H A(I2,5H)  =   F10.5,5X,3H B(I2,5H)  =   F10.5)
      GO TO 18
      END
```

```
VTF COEFFICIENT EVALUATION, INPUT DATA, ZOBEL FILTER, M=0.6
.0  .6  .0  .0  .0  .0    1.  .0  .64  .0  .0  .0
1.6  2.  1.6
1.  .6  .64    1.  .0  .64


VTF COEFFICIENT EVALUATION, INPUT DATA, ZOBEL FILTER, M=0.707
.0  .70710678  .0  .0  .0  .0    1.  .0  .5  .0  .0  .0
1.70710678  2.  1.70710678
1.  .70710678  .5    1.  .0  .5


VTF COEFFICIENT EVALUATION, INPUT DATA, TYPE 1 FILTER
.0  2.  2.  1.  .0  .0    2.  2.  2.  1.  .0  .0
1.  2.  1.
1.  .0  .0    1.  .0  .0


VTF COEFFICIENT EVALUATION, INPUT DATA, TYPE 2 FILTER
.0  2.  2.  1.  .0  .0    3.236  3.236  2.618  1.  .0  .0
1.618  2.  1.
1.  .0  .0    1.  .0  .0


VTF COEFFICIENT EVALUATION, INPUT DATA, TYPE 3 FILTER
.0  2.155  2.155  1.  .0  .0    3.7325  3.7325  2.732  1.  .0  .0
1.732  2.155  1.
1.  .0  .0    1.  .0  .C


VTF COEFFICIENT EVALUATION, INPUT DATA, TYPE 4 FILTER
.0  .665  1.33  1.  .0  .0    .9975  1.995  2.  1.  .0  .0
1.5  1.333  .5
1.  .0  .0    1.  .0  .0


VTF COEFFICIENT EVALUATION, INPUT DATA, TYPE 6 FILTER
.0  2.0         3.2360679  3.8541019  2.61803399  1.
3.2360679  5.2360679  7.23606789 5.8541019  3.2360679  1.
1.61803399  2.  1.61803399
1.  .61803399  .61803399    1.  .0  .61803399


VTF COEFFICIENT EVALUATION, INPUT DATA, TYPE 7 FILTER
.0  .8395616  1.7143309  1.9662335  1.3674296  .51436894
1.3247343  2.7050226  3.5343409  3.0394495  1.6934152  .51436894
1.5778883  1.9441298  1.7143309
1.  .7143309  .4897314    1.  .0  .4897314
```

```
C   C   VTF COEFFICIENT EVALUATION, OUTPUT DATA, ZOBEL FILTER, FIG.22
        A( 0)  =     1.00000        B( 0)  =     2.00000
        A( 1)  =     0.00000        B( 1)  =     6.40000
        A( 2)  =     1.28000        B( 2)  =    12.80000
        A( 3)  =     0.00000        B( 3)  =    17.53600
        A( 4)  =      .40960        B( 4)  =    17.61280
        A( 5)  =     0.00000        B( 5)  =    12.98432
        A( 6)  =     0.00000        B( 6)  =     6.55360
        A( 7)  =     0.00000        B( 7)  =     2.09715
        A( 8)  =     0.00000        B( 8)  =     0.00000
        A( 9)  =     0.00000        B( 9)  =     0.00000
        A(10)  =     0.00000        B(10)  =     0.00000

C   C   VTF COEFFICIENT EVALUATION, OUTPUT DATA, ZOBEL FILTER, FIG.25
        A( 0)  =     1.00000        B( 0)  =     2.00000
        A( 1)  =     0.00000        B( 1)  =     6.82843
        A( 2)  =     1.00000        B( 2)  =    13.65685
        A( 3)  =     0.00000        B( 3)  =    18.48528
        A( 4)  =      .25000        B( 4)  =    17.98528
        A( 5)  =     0.00000        B( 5)  =    12.51041
        A( 6)  =     0.00000        B( 6)  =     5.82843
        A( 7)  =     0.00000        B( 7)  =     1.45711
        A( 8)  =     0.00000        B( 8)  =     0.00000
        A( 9)  =     0.00000        B( 9)  =     0.00000
        A(10)  =     0.00000        B(10)  =     0.00000

C   C   VTF COEFFICIENT EVALUATION, OUTPUT DATA, TYPE 1 FILTER, FIG.28
        A( 0)  =     2.00000        B( 0)  =     4.00000
        A( 1)  =     2.00000        B( 1)  =    14.00000
        A( 2)  =     2.00000        B( 2)  =    26.00000
        A( 3)  =     1.00000        B( 3)  =    31.00000
        A( 4)  =     0.00000        B( 4)  =    26.00000
        A( 5)  =     0.00000        B( 5)  =    14.00000
        A( 6)  =     0.00000        B( 6)  =     4.00000
        A( 7)  =     0.00000        B( 7)  =     0.00000
        A( 8)  =     0.00000        B( 8)  =     0.00000
        A( 9)  =     0.00000        B( 9)  =     0.00000
        A(10)  =     0.00000        B(10)  =     0.00000

C   C   VTF COEFFICIENT EVALUATION, OUTPUT DATA, TYPE 2 FILTER, FIG.31
        A( 0)  =     3.23600        B( 0)  =     6.47200
        A( 1)  =     3.23600        B( 1)  =    23.41585
        A( 2)  =     2.61800        B( 2)  =    44.35954
        A( 3)  =     1.00000        B( 3)  =    54.21332
        A( 4)  =     0.00000        B( 4)  =    44.35954
        A( 5)  =     0.00000        B( 5)  =    23.41585
        A( 6)  =     0.00000        B( 6)  =     6.47200
        A( 7)  =     0.00000        B( 7)  =     0.00000
        A( 8)  =     0.00000        B( 8)  =     0.00000
        A( 9)  =     0.00000        B( 9)  =     0.00000
        A(10)  =     0.00000        B(10)  =     0.00000
```

```
C   C   VTF COEFFICIENT EVALUATION, OUTPUT DATA, TYPE 3 FILTER, FIG.34
        A( 0)   =     3.73250       B( 0)   =     7.46500
        A( 1)   =     3.73250       B( 1)   =    27.86073
        A( 2)   =     2.73200       B( 2)   =    53.72213
        A( 3)   =     1.00000       B( 3)   =    66.18855
        A( 4)   =     0.00000       B( 4)   =    53.72185
        A( 5)   =     0.00000       B( 5)   =    27.86045
        A( 6)   =     0.00000       B( 6)   =     7.46492
        A( 7)   =     0.00000       B( 7)   =     0.00000
        A( 8)   =     0.00000       B( 8)   =     0.00000
        A( 9)   =     0.00000       B( 9)   =     0.00000
        A(10)   =     0.00000       B(10)   =     0.00000

C   C   VTF COEFFICIENT EVALUATION, OUTPUT DATA, TYPE 4 FILTER, FIG.37
        A( 0)   =      .99750       B( 0)   =     1.99500
        A( 1)   =     1.99500       B( 1)   =     7.97967
        A( 2)   =     2.00000       B( 2)   =    15.96867
        A( 3)   =     1.00000       B( 3)   =    19.97159
        A( 4)   =     0.00000       B( 4)   =    15.98367
        A( 5)   =     0.00000       B( 5)   =     7.99467
        A( 6)   =     0.00000       B( 6)   =     1.99950
        A( 7)   =     0.00000       B( 7)   =     0.00000
        A( 8)   =     0.00000       B( 8)   =     0.00000
        A( 9)   =     0.00000       B( 9)   =     0.00000
        A(10)   =     0.00000       B(10)   =     0.00000

C   C   VTF COEFFICIENT EVALUATION, OUTPUT DATA, TYPE 6 FILTER, FIG.42
        A( 0)   =     3.23607       B( 0)   =     6.47214
        A( 1)   =     5.23607       B( 1)   =    31.41641
        A( 2)   =     9.23607       B( 2)   =    86.24922
        A( 3)   =     9.09017       B( 3)   =   164.82465
        A( 4)   =     7.70820       B( 4)   =   237.74764
        A( 5)   =     4.61803       B( 5)   =   267.83781
        A( 6)   =     2.00000       B( 6)   =   237.74766
        A( 7)   =      .61803       B( 7)   =   164.82468
        A( 8)   =     0.00000       B( 8)   =    86.24923
        A( 9)   =     0.00000       B( 9)   =    31.41641
        A(10)   =     0.00000       B(10)   =     6.47214

C   C   VTF COEFFICIENT EVALUATION, OUTPUT DATA, TYPE 7 FILTER, FIG.45
        A( 0)   =     1.32473       B( 0)   =     2.64947
        A( 1)   =     2.70502       B( 1)   =    14.13268
        A( 2)   =     4.18310       B( 2)   =    40.53567
        A( 3)   =     4.36418       B( 3)   =    79.52404
        A( 4)   =     3.42429       B( 4)   =   116.33964
        A( 5)   =     2.00288       B( 5)   =   131.63155
        A( 6)   =      .82932       B( 6)   =   116.33964
        A( 7)   =      .25190       B( 7)   =    79.52404
        A( 8)   =     0.00000       B( 8)   =    40.53567
        A( 9)   =     0.00000       B( 9)   =    14.13268
        A(10)   =     0.00000       B(10)   =     2.64947
```

APPENDIX H

FORGO Programs for Sinusoidal Response Calculations of the VTF. To make possible the computation of the sinusoidal VTF response, the FORGO digital computer program shown on pages 102 and 103 was written and used. The input and output data for each of the previously discussed filter configurations, except Type 5, are given on the pages following the computer program.

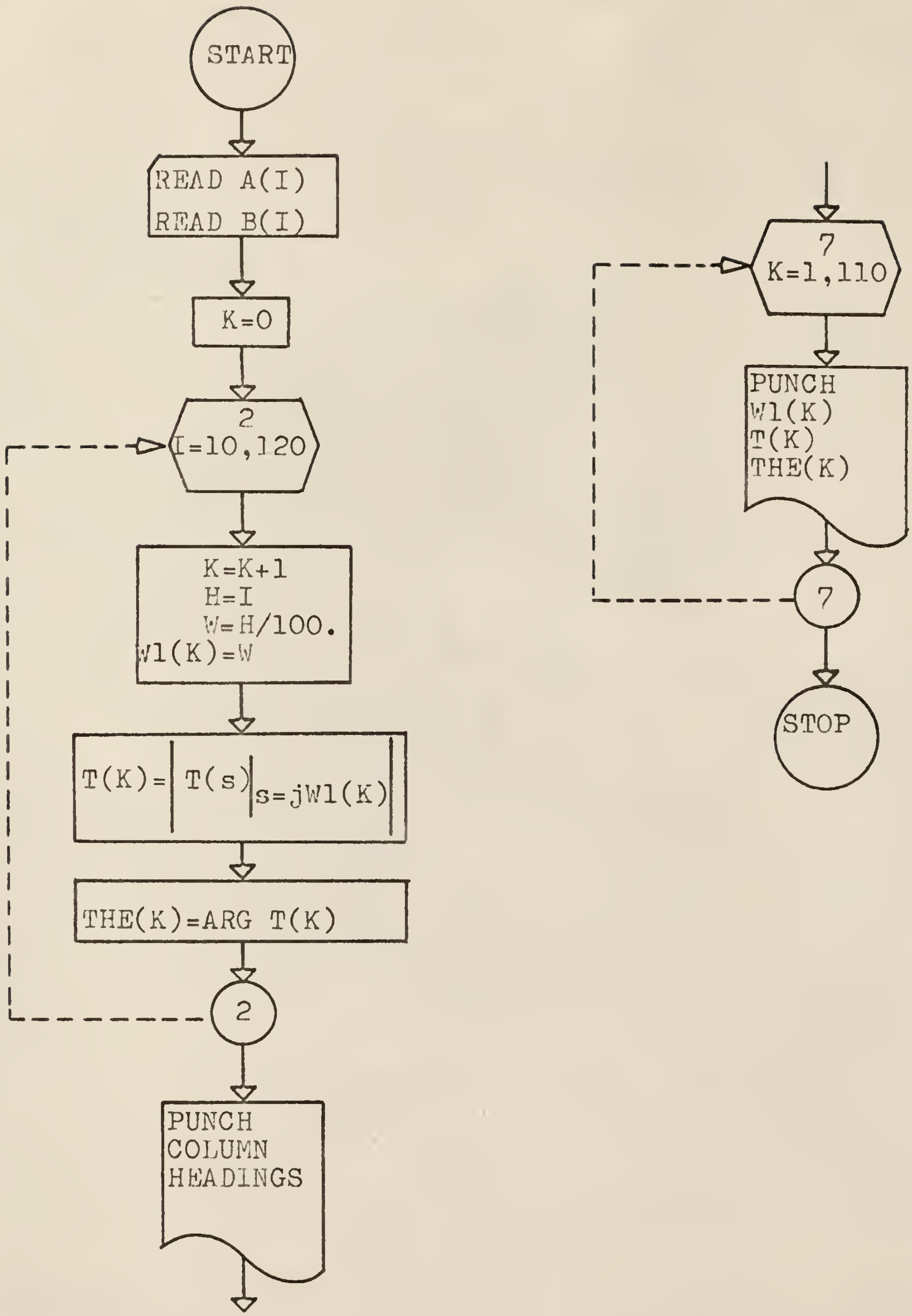

Fig. 58. Block diagram of VTF computer program.

```
C   C   VTF( T(S) - PHASE SHIFT ) VS. FREQUENCY, ALL FILTERS
        DIMENSION A(7),B(10),W1(112),T(112),THE(112)
      4 FORMAT(/7X,2HW1,5X,5HT(JW),6X,2HO1,9X,2HW1,5X,5HT(JW),6X,2HO1//)
      5 FORMAT( 2(5X,F5.2,3X,F7.5,2X,F7.2))
     45 READ,A0,(A(I),I=1,7),B0,(B(I),I=1,10)
        K=0
        DO 2 I=10,120
        K=K+1
        H=I
        W=H/100.
        W1(K)=W
        C=A0-A(2)*W**2+A(4)*W**4-A(6)*W**6
        D=A(1)*W-A(3)*W**3+A(5)*W**5-A(7)*W**7
        E=B0-B(2)*W**2+B(4)*W**4-B(6)*W**6+B(8)*W**8-B(10)*W**10
        F=B(1)*W-B(3)*W**3+B(5)*W**5-B(7)*W**7+B(9)*W**9
        T(K)=SQRT((C**2+D**2)/(E**2+F**2))
        IF(ABS(C)-.1E-40)3,3,8
      3 THE1=0.
        GO TO 14
      8 IF(C)10,10,11
     11 IF(D)12,12,13
     13 THE1=ATAN(D/C)*57.245779
        GO TO 14
     12 THE1=ATAN(D/C)*57.24779
        GO TO 14
     10 IF(D)15,15,16
     16 THE1=180.-ATAN(D/ABS(C))*57.245779
        GO TO 14
     15 THE1=180.+ATAN(D/C)*57.245779
     14 IF(ABS(E)-.1E-40)9,9,28
      9 THE2=0.
        GO TO 20
     28 IF(E)19,19,21
     21 IF(F)22,22,23
     23 THE2=ATAN(F/E)*57.245779
        GO TO 20
     22 THE2=ATAN(F/E)*57.245779
        GO TO 20
     19 IF(F)24,24,25
     25 THE2=180.-ATAN(F/ABS(E))*57.245779
        GO TO 20
     24 THE2=180.+ATAN(F/E)*57.245779
     20 THE(K)=THE1-THE2
      2 CONTINUE
        PUNCH 4
        DO 7 J=1,55
        PUNCH 5,W1(J),T(J),THE(J),W1(J+55),T(J+55),THE(J+55)
      7 CONTINUE
        GO TO 45
        END
```

```
VTF COEFFICIENTS, INPUT DATA, CONSTANT-1 FILTER
1.  .0  .0  .0  .0  .0  .0  .0
2.  4.  4.  2.  .0  .0  .0  .0  .0  .0  .0

VTF COEFFICIENTS, INPUT DATA, ZOBEL FILTER, M=0.6
1.  .0  1.28  .0  .4096  .0  .0  .0
2.  6.4  12.8  17.536  17.6128  12.98432  6.5536  2.09715  .0  .0  .0

VTF COEFFICIENTS, INPUT DATA, ZOBEL FILTER, M=0.707
1.  .0  1.  .0  .25  .0  .0  .0
2.  6.82843  13.65685  18.48528  17.98528  12.51041  5.82843  1.45711
.0  .0  .0

VTF COEFFICIENTS, INPUT DATA, TYPE 1 FILTER
2.  2.  2.  1.  .0  .0  .0  .0
4.  14.  26.  31.  26.  14.  4.  .0  .0  .0  .0

VTF COEFFICIENTS, INPUT DATA, TYPE 2 FILTER
3.236  3.236  2.618  1.  .0  .0  .0  .0
6.472  23.41585  44.35954  54.21332  44.35954  23.41585  6.472  .0  .0
.0  .0

VTF COEFFICIENTS, INPUT DATA, TYPE 3 FILTER
3.7325  3.7325  2.732  1.  .0  .0  .0  .0
7.465  27.86073  53.72213  66.18855  53.72185  27.86045  7.46492  .0
.0  .0  .0

VTF COEFFICIENTS, INPUT DATA, TYPE 4 FILTER
.9975  1.995  2.  1.  .0  .0  .0  .0
1.995  7.97967  15.96867  19.97159  15.98367  7.99467  1.9995  .0  .0
.0  .0

VTF COEFFICIENTS, INPUT DATA, TYPE 6 FILTER
3.2360679 5.2360679 9.2360679 9.0901699 7.7082039 4.6180339 2. .61803398
6.4721359 31.416407 86.249223 164.82468 237.74767 267.83784 237.74767
164.82468 86.249223 31.416407 6.4721359

VTF COEFFICIENTS, INPUT DATA, TYPE 7 FILTER
      1.85451084      3.78679208      5.85597666      6.10947126
4.79370693
      2.80385847      1.16097257      0.35264130
      3.7090216      19.7844959      56.7463506      111.3266175
162.8651935
    184.2725252     162.8651935     111.3266175      56.7463506
19.7844959
      3.7090216
```

C C VTF(T(S)-PHASE) VS. FREQUENCY, OUTPUT DATA, CONSTANT-1 FILTER, FIG.19

W1	T(JW)	O1	W1	T(JW)	O1
.10	.50000	-11.47	.65	.48215	-81.33
.11	.50000	-12.62	.66	.48054	-82.82
.12	.50000	-13.77	.67	.47881	-84.31
.14	.50000	-16.08	.69	.47502	-87.32
.15	.50000	-17.24	.70	.47295	-88.84
.17	.49999	-19.56	.72	.46843	-92.05
.19	.49999	-21.89	.74	.46340	-95.14
.20	.49998	-23.06	.75	.46068	-96.69
.22	.49997	-25.40	.77	.45484	-99.80
.23	.49996	-26.58	.78	.45172	-101.36
.24	.49995	-27.76	.79	.44846	-102.93
.25	.49994	-28.94	.80	.44506	-104.50
.26	.49992	-30.13	.81	.44152	-106.07
.27	.49990	-31.32	.82	.43785	-107.64
.28	.49988	-32.51	.83	.43405	-109.21
.29	.49985	-33.71	.84	.43012	-110.78
.30	.49982	-34.91	.85	.42607	-112.34
.31	.49978	-36.12	.86	.42189	-113.91
.32	.49973	-37.33	.87	.41759	-115.47
.33	.49968	-38.55	.88	.41318	-117.02
.34	.49961	-39.77	.89	.40866	-118.57
.35	.49954	-41.00	.90	.40404	-120.12
.36	.49946	-42.23	.91	.39931	-121.66
.37	.49936	-43.47	.92	.39450	-123.19
.38	.49925	-44.72	.93	.38960	-124.71
.39	.49912	-45.97	.94	.38463	-126.22
.40	.49898	-47.22	.95	.37958	-127.72
.41	.49882	-48.49	.96	.37448	-129.21
.42	.49863	-49.76	.97	.36931	-130.68
.43	.49843	-51.04	.98	.36410	-132.15
.44	.49820	-52.32	.99	.35884	-133.60
.45	.49794	-53.62	1.00	.35355	-135.04
.46	.49765	-54.92	1.01	.34824	-136.46
.47	.49733	-56.23	1.02	.34290	-137.87
.48	.49697	-57.54	1.03	.33756	-139.27
.49	.49658	-58.87	1.04	.33220	-140.64
.50	.49614	-60.20	1.05	.32685	-142.01
.51	.49566	-61.55	1.06	.32151	-143.35
.52	.49513	-62.90	1.07	.31618	-144.68
.53	.49455	-64.26	1.08	.31087	-145.99
.54	.49391	-65.63	1.09	.30559	-147.29
.55	.49322	-67.01	1.10	.30034	-148.56
.56	.49246	-68.40	1.11	.29512	-149.82
.57	.49164	-69.80	1.12	.28994	-151.06
.58	.49075	-71.21	1.13	.28481	-152.29
.59	.48978	-72.62	1.14	.27973	-153.49
.60	.48873	-74.05	1.15	.27470	-154.68
.61	.48760	-75.49	1.16	.26972	-155.85
.62	.48638	-76.94	1.17	.26481	-157.00
.63	.48507	-78.39	1.18	.25995	-158.13
.64	.48366	-79.86	1.19	.25516	-159.25

C C VTF(T(S)-PHASE) VS. FREQUENCY, OUTPUT DATA, ZOBEL FILTER, FIG.22

W1	T(JW)	O1	W1	T(JW)	O1
.10	.50000	-18.36	.65	.49980	-135.43
.11	.50000	-20.21	.66	.49981	-138.20
.12	.50000	-22.06	.67	.49984	-141.02
.14	.50000	-25.77	.69	.49988	-146.80
.15	.50000	-27.63	.70	.49990	-149.78
.17	.50000	-31.37	.72	.49994	-155.92
.19	.50000	-35.12	.74	.49997	-162.33
.20	.50000	-37.01	.75	.49998	-165.64
.22	.50000	-40.80	.77	.50000	-172.53
.23	.49999	-42.70	.78	.50000	-176.11
.24	.49999	-44.61	.79	.50000	-179.79
.25	.49999	-46.53	.80	.50000	-183.58
.26	.49999	-48.45	.81	.50000	-187.49
.27	.49999	-50.39	.82	.50000	-191.52
.28	.49998	-52.33	.83	.50000	-195.70
.29	.49998	-54.28	.84	.50000	-200.04
.30	.49997	-56.24	.85	.49998	-204.55
.31	.49997	-58.21	.86	.49994	-209.26
.32	.49996	-60.19	.87	.49984	-214.18
.33	.49996	-62.18	.88	.49964	-219.35
.34	.49995	-64.18	.89	.49924	-224.81
.35	.49994	-66.19	.90	.49852	-230.59
.36	.49994	-68.21	.91	.49727	-236.74
.37	.49993	-70.25	.92	.49516	-243.33
.38	.49992	-72.30	.93	.49174	-250.40
.39	.49991	-74.36	.94	.48635	-258.03
.40	.49990	-76.44	.95	.47811	-266.27
.41	.49989	-78.53	.96	.46595	84.68
.42	.49987	-80.64	.97	.44873	75.15
.43	.49986	-82.76	.98	.42551	65.05
.44	.49985	-84.90	.99	.39596	54.52
.45	.49984	-87.06	1.00	.36070	43.79
.46	.49982	-89.23	1.01	.32136	33.14
.47	.49981	-91.58	1.02	.28030	22.81
.48	.49980	-93.79	1.03	.23990	13.03
.49	.49978	-96.03	1.04	.20210	3.90
.50	.49977	-98.28	1.05	.16809	-4.52
.51	.49976	-100.56	1.06	.13837	-12.24
.52	.49975	-102.86	1.07	.11293	-19.30
.53	.49974	-105.18	1.08	.09149	-25.78
.54	.49973	-107.53	1.09	.07361	-31.74
.55	.49973	-109.91	1.10	.05880	-37.24
.56	.49972	-112.31	1.11	.04661	-42.34
.57	.49972	-114.74	1.12	.03663	-47.09
.58	.49972	-117.20	1.13	.02850	-51.53
.59	.49973	-119.70	1.14	.02191	-55.70
.60	.49973	-122.23	1.15	.01659	-59.62
.61	.49974	-124.79	1.16	.01234	-63.34
.62	.49975	-127.39	1.17	.00896	-66.86
.63	.49976	-130.03	1.18	.00632	-70.21
.64	.49978	-132.71	1.19	.00429	-73.40

C C VTF(T(S)-PHASE) VS. FREQUENCY, OUTPUT DATA, ZOBEL FILTER, FIG.25

W1	T(JW)	O1	W1	T(JW)	O1
.10	.50000	-19.59	.65	.49988	-143.43
.11	.50000	-21.56	.66	.49987	-146.31
.12	.50000	-23.53	.67	.49987	-149.23
.14	.50000	-27.49	.69	.49988	-155.22
.15	.50000	-29.47	.70	.49989	-158.29
.17	.50000	-33.46	.72	.49993	-164.62
.19	.50000	-37.46	.74	.49997	-171.21
.20	.50000	-39.47	.75	.49999	-174.61
.22	.50000	-43.50	.77	.50000	-181.67
.23	.50000	-45.53	.78	.49998	-185.33
.24	.50000	-47.56	.79	.49992	-189.10
.25	.50000	-49.61	.80	.49981	-192.97
.26	.50000	-51.65	.81	.49961	-196.97
.27	.50000	-53.71	.82	.49930	-201.10
.28	.50000	-55.78	.83	.49882	-205.38
.29	.50000	-57.85	.84	.49811	-209.81
.30	.50000	-59.94	.85	.49709	-214.42
.31	.50000	-62.03	.86	.49565	-219.21
.32	.50000	-64.13	.87	.49366	-224.22
.33	.50000	-66.25	.88	.49093	-229.44
.34	.50000	-68.37	.89	.48727	-234.91
.35	.50000	-70.51	.90	.48242	-240.63
.36	.50000	-72.66	.91	.47609	-246.61
.37	.50000	-74.82	.92	.46799	-252.86
.38	.50000	-77.00	.93	.45780	-259.38
.39	.50000	-79.18	.94	.44523	-266.15
.40	.50000	-81.39	.95	.43009	86.70
.41	.50000	-83.60	.96	.41228	79.53
.42	.49999	-85.84	.97	.39191	72.23
.43	.49999	-88.08	.98	.36924	64.88
.44	.49999	-90.50	.99	.34473	57.55
.45	.49999	-92.79	1.00	.31898	50.32
.46	.49999	-95.09	1.01	.29264	43.24
.47	.49998	-97.41	1.02	.26638	36.38
.48	.49998	-99.74	1.03	.24077	29.78
.49	.49998	-102.10	1.04	.21629	23.46
.50	.49997	-104.48	1.05	.19328	17.45
.51	.49997	-106.88	1.06	.17194	11.73
.52	.49997	-109.31	1.07	.15237	6.32
.53	.49996	-111.75	1.08	.13459	1.19
.54	.49995	-114.23	1.09	.11856	-3.68
.55	.49995	-116.73	1.10	.10418	-8.29
.56	.49994	-119.25	1.11	.09133	-12.66
.57	.49993	-121.81	1.12	.07991	-16.82
.58	.49993	-124.39	1.13	.06976	-20.78
.59	.49992	-127.00	1.14	.06079	-24.56
.60	.49991	-129.65	1.15	.05285	-28.17
.61	.49990	-132.33	1.16	.04585	-31.62
.62	.49989	-135.05	1.17	.03968	-34.94
.63	.49989	-137.80	1.18	.03424	-38.12
.64	.49988	-140.59	1.19	.02947	-41.17

C C VTF(T(S)-PHASE) VS. FREQUENCY, OUTPUT DATA, TYPE 1 FILTER, FIG.28

W1	T(JW)	O1	W1	T(JW)	O1
.10	.49935	-14.34	.65	.44796	-98.59
.11	.49920	-15.78	.66	.44641	-100.23
.12	.49904	-17.23	.67	.44488	-101.89
.14	.49866	-20.12	.69	.44192	-105.23
.15	.49845	-21.57	.70	.44051	-106.93
.17	.49795	-24.48	.72	.43781	-110.36
.19	.49737	-27.40	.74	.43532	-113.87
.20	.49704	-28.86	.75	.43417	-115.66
.22	.49631	-31.80	.77	.43202	-119.34
.23	.49590	-33.27	.78	.43102	-121.23
.24	.49547	-34.75	.79	.43007	-123.16
.25	.49501	-36.23	.80	.42914	-125.14
.26	.49451	-37.71	.81	.42824	-127.16
.27	.49398	-39.20	.82	.42732	-129.25
.28	.49342	-40.69	.83	.42638	-131.39
.29	.49283	-42.19	.84	.42536	-133.61
.30	.49220	-43.68	.85	.42422	-135.90
.31	.49154	-45.19	.86	.42290	-138.28
.32	.49084	-46.69	.87	.42134	-140.74
.33	.49010	-48.20	.88	.41945	-143.30
.34	.48932	-49.72	.89	.41713	-145.96
.35	.48850	-51.23	.90	.41427	-148.72
.36	.48764	-52.75	.91	.41075	-151.58
.37	.48675	-54.28	.92	.40644	-154.54
.38	.48581	-55.81	.93	.40121	-157.60
.39	.48483	-57.34	.94	.39496	-160.74
.40	.48381	-58.88	.95	.38758	-163.94
.41	.48275	-60.42	.96	.37900	-167.19
.42	.48164	-61.96	.97	.36923	-170.45
.43	.48050	-63.50	.98	.35828	-173.70
.44	.47932	-65.21	.99	.34627	-176.89
.45	.47809	-66.76	1.00	.33333	0.00
.46	.47683	-68.32	1.01	.31967	177.01
.47	.47553	-69.88	1.02	.30552	174.18
.48	.47419	-71.44	1.03	.29111	171.53
.49	.47281	-73.01	1.04	.27669	169.08
.50	.47140	-74.58	1.05	.26247	166.85
.51	.46996	-76.15	1.06	.24863	164.85
.52	.46849	-77.73	1.07	.23534	163.08
.53	.46699	-79.31	1.08	.22270	161.54
.54	.46547	-80.89	1.09	.21078	160.21
.55	.46392	-82.48	1.10	.19963	159.08
.56	.46236	-84.07	1.11	.18927	158.15
.57	.46077	-85.66	1.12	.17969	157.39
.58	.45918	-87.26	1.13	.17086	156.78
.59	.45757	-88.86	1.14	.16275	156.32
.60	.45596	-90.47	1.15	.15531	155.97
.61	.45434	-92.08	1.16	.14851	155.72
.62	.45273	-93.69	1.17	.14229	155.55
.63	.45113	-95.32	1.18	.13659	155.46
.64	.44954	-96.95	1.19	.13138	155.41

C C VTF(T(S)-PHASE) VS. FREQUENCY, OUTPUT DATA, TYPE 2 FILTER, FIG.31

W1	T(JW)	O1	W1	T(JW)	O1
.10	.50000	-15.01	.65	.49079	-107.20
.11	.50000	-16.52	.66	.48988	-109.24
.12	.50000	-18.03	.67	.48888	-111.31
.14	.50000	-21.06	.69	.48660	-115.51
.15	.49999	-22.57	.70	.48529	-117.66
.17	.49999	-25.61	.72	.48230	-122.04
.19	.49998	-28.66	.74	.47871	-126.54
.20	.49998	-30.19	.75	.47666	-128.84
.22	.49997	-33.27	.77	.47194	-133.55
.23	.49996	-34.81	.78	.46923	-135.96
.24	.49995	-36.35	.79	.46627	-138.41
.25	.49994	-37.90	.80	.46303	-140.90
.26	.49993	-39.46	.81	.45948	-143.42
.27	.49992	-41.02	.82	.45561	-145.98
.28	.49990	-42.58	.83	.45139	-148.58
.29	.49989	-44.15	.84	.44678	-151.21
.30	.49986	-45.73	.85	.44178	-153.88
.31	.49984	-47.31	.86	.43636	-156.59
.32	.49981	-48.90	.87	.43049	-159.32
.33	.49978	-50.49	.88	.42416	-162.08
.34	.49975	-52.09	.89	.41736	-164.87
.35	.49970	-53.70	.90	.41009	-167.68
.36	.49966	-55.31	.91	.40233	-170.50
.37	.49961	-56.93	.92	.39410	-173.33
.38	.49955	-58.56	.93	.38540	-176.16
.39	.49948	-60.20	.94	.37627	-178.99
.40	.49941	-61.85	.95	.36672	-181.81
.41	.49932	-63.50	.96	.35679	-184.61
.42	.49923	-65.32	.97	.34653	-187.38
.43	.49913	-67.00	.98	.33599	-190.11
.44	.49901	-68.68	.99	.32522	-192.80
.45	.49888	-70.37	1.00	.31427	74.48
.46	.49874	-72.08	1.01	.30322	161.83
.47	.49858	-73.80	1.02	.29211	159.32
.48	.49841	-75.52	1.03	.28102	156.88
.49	.49822	-77.26	1.04	.26999	154.53
.50	.49800	-79.01	1.05	.25909	152.25
.51	.49777	-80.78	1.06	.24837	150.07
.52	.49751	-82.56	1.07	.23786	147.98
.53	.49723	-84.35	1.08	.22760	145.98
.54	.49692	-86.16	1.09	.21764	144.08
.55	.49658	-87.98	1.10	.20799	142.27
.56	.49620	-89.82	1.11	.19868	140.57
.57	.49579	-91.68	1.12	.18973	139.12
.58	.49535	-93.55	1.13	.18114	137.60
.59	.49486	-95.44	1.14	.17291	136.19
.60	.49432	-97.35	1.15	.16507	134.87
.61	.49373	-99.28	1.16	.15759	133.64
.62	.49309	-101.23	1.17	.15047	132.50
.63	.49239	-103.20	1.18	.14372	131.44

C C VTF(T(S)-PHASE) VS. FREQUENCY, OUTPUT DATA, TYPE 3 FILTER, FIG.34

W1	T(JW)	O1	W1	T(JW)	O1
.10	.50000	-15.67	.65	.48203	-113.19
.11	.50000	-17.25	.66	.48023	-115.37
.12	.50000	-18.82	.67	.47828	-117.58
.14	.50000	-21.98	.69	.47390	-122.06
.15	.50000	-23.57	.70	.47144	-124.34
.17	.49999	-26.74	.72	.46594	-128.98
.19	.49999	-29.93	.74	.45957	-133.72
.20	.49999	-31.53	.75	.45604	-136.13
.22	.49998	-34.75	.77	.44821	-141.02
.23	.49997	-36.36	.78	.44389	-143.50
.24	.49996	-37.98	.79	.43929	-146.01
.25	.49995	-39.61	.80	.43440	-148.54
.26	.49993	-41.23	.81	.42921	-151.09
.27	.49992	-42.87	.82	.42371	-153.66
.28	.49990	-44.51	.83	.41789	-156.25
.29	.49987	-46.16	.84	.41176	-158.85
.30	.49984	-47.81	.85	.40531	-161.47
.31	.49981	-49.47	.86	.39854	-164.10
.32	.49977	-51.14	.87	.39145	-166.74
.33	.49972	-52.82	.88	.38406	-169.38
.34	.49967	-54.50	.89	.37638	-172.03
.35	.49960	-56.19	.90	.36841	-174.67
.36	.49953	-57.90	.91	.36017	-177.31
.37	.49944	-59.61	.92	.35168	-179.94
.38	.49934	-61.33	.93	.34296	-182.56
.39	.49923	-63.05	.94	.33404	-185.15
.40	.49910	-64.79	.95	.32495	-187.73
.41	.49895	-66.70	.96	.31572	-190.27
.42	.49879	-68.46	.97	.30638	-192.79
.43	.49860	-70.24	.98	.29695	-195.26
.44	.49839	-72.02	.99	.28748	-197.70
.45	.49815	-73.82	1.00	.27800	-200.09
.46	.49788	-75.63	1.01	.26854	157.41
.47	.49758	-77.46	1.02	.25913	155.12
.48	.49725	-79.29	1.03	.24980	152.89
.49	.49688	-81.15	1.04	.24059	150.71
.50	.49646	-83.01	1.05	.23152	148.60
.51	.49601	-84.90	1.06	.22262	146.55
.52	.49550	-86.80	1.07	.21390	144.57
.53	.49494	-88.71	1.08	.20539	142.65
.54	.49432	-90.64	1.09	.19710	140.81
.55	.49363	-92.60	1.10	.18904	139.04
.56	.49288	-94.56	1.11	.18124	137.34
.57	.49206	-96.55	1.12	.17369	135.71
.58	.49115	-98.56	1.13	.16640	134.15
.59	.49016	-100.58	1.14	.15938	132.66
.60	.48908	-102.63	1.15	.15263	131.25
.61	.48790	-104.70	1.16	.14615	129.91
.62	.48661	-106.79	1.17	.13994	128.80
.63	.48521	-108.90	1.18	.13399	127.59
.64	.48368	-111.03	1.19	.12830	126.46

C C VTF(T(S)-PHASE) VS. FREQUENCY, OUTPUT DATA, TYPE 4 FILTER, FIG.37

W1	T(JW)	O1	W1	T(JW)	O1
.10	.50000	-11.47	.65	.48203	-81.49
.11	.50000	-12.62	.66	.48040	-82.98
.12	.50000	-13.77	.67	.47867	-84.47
.14	.50000	-16.08	.69	.47486	-87.48
.15	.50000	-17.24	.70	.47278	-89.00
.17	.49999	-19.56	.72	.46824	-91.89
.19	.49999	-21.89	.74	.46318	-94.98
.20	.49998	-23.06	.75	.46046	-96.52
.22	.49997	-25.40	.77	.45460	-99.64
.23	.49996	-26.58	.78	.45146	-101.20
.24	.49995	-27.76	.79	.44819	-102.76
.25	.49994	-28.94	.80	.44479	-104.33
.26	.49992	-30.13	.81	.44124	-105.90
.27	.49990	-31.32	.82	.43757	-107.46
.28	.49988	-32.51	.83	.43376	-109.03
.29	.49985	-33.71	.84	.42983	-110.60
.30	.49982	-34.91	.85	.42577	-112.16
.31	.49978	-36.12	.86	.42159	-113.72
.32	.49973	-37.33	.87	.41729	-115.28
.33	.49968	-38.55	.88	.41288	-116.84
.34	.49961	-39.77	.89	.40836	-118.38
.35	.49954	-41.00	.90	.40374	-119.92
.36	.49945	-42.23	.91	.39903	-121.46
.37	.49936	-43.47	.92	.39422	-122.99
.38	.49924	-44.71	.93	.38933	-124.50
.39	.49912	-46.12	.94	.38436	-126.01
.40	.49897	-47.38	.95	.37933	-127.51
.41	.49881	-48.64	.96	.37423	-129.00
.42	.49862	-49.91	.97	.36907	-130.47
.43	.49842	-51.19	.98	.36387	-131.93
.44	.49818	-52.48	.99	.35863	-133.38
.45	.49792	-53.77	1.00	.35336	225.03
.46	.49763	-55.07	1.01	.34805	223.60
.47	.49731	-56.38	1.02	.34273	222.20
.48	.49695	-57.70	1.03	.33740	220.81
.49	.49655	-59.03	1.04	.33207	219.43
.50	.49611	-60.36	1.05	.32673	218.07
.51	.49563	-61.70	1.06	.32140	216.73
.52	.49510	-63.06	1.07	.31609	215.40
.53	.49451	-64.42	1.08	.31080	214.09
.54	.49387	-65.79	1.09	.30553	212.80
.55	.49317	-67.17	1.10	.30029	211.52
.56	.49241	-68.56	1.11	.29509	210.26
.57	.49158	-69.96	1.12	.28993	209.02
.58	.49068	-71.36	1.13	.28481	207.80
.59	.48971	-72.78	1.14	.27974	206.60
.60	.48865	-74.21	1.15	.27472	205.41
.61	.48751	-75.65	1.16	.26976	204.24
.62	.48628	-77.09	1.17	.26486	203.09
.63	.48496	-78.55	1.18	.26001	201.95
.64	.48355	-80.02	1.19	.25524	200.84

C C VTF(T(S)-PHASE) VS. FREQUENCY, OUTPUT DATA, TYPE 6 FILTER, FIG.42

W1	T(JW)	O1	W1	T(JW)	O1
.10	.50000	-18.57	.65	.49916	-134.78
.11	.50000	-20.43	.66	.49913	-137.45
.12	.50000	-22.30	.67	.49910	-140.16
.14	.50000	-26.05	.69	.49904	-145.71
.15	.50000	-27.92	.70	.49902	-148.56
.17	.50000	-31.69	.72	.49897	-154.41
.19	.50000	-35.48	.74	.49892	-160.51
.20	.50000	-37.38	.75	.49888	-163.66
.22	.50000	-41.19	.77	.49877	-170.19
.23	.50000	-43.11	.78	.49867	-173.58
.24	.49999	-45.03	.79	.49854	-176.91
.25	.49999	-46.96	.80	.49836	179.35
.26	.49999	-48.89	.81	.49810	175.65
.27	.49999	-50.84	.82	.49775	171.83
.28	.49999	-52.79	.83	.49727	167.88
.29	.49998	-54.74	.84	.49662	163.79
.30	.49998	-56.71	.85	.49575	159.53
.31	.49998	-58.68	.86	.49461	155.09
.32	.49997	-60.82	.87	.49309	150.46
.33	.49997	-62.82	.88	.49110	145.61
.34	.49996	-64.82	.89	.48846	140.52
.35	.49995	-66.83	.90	.48502	135.16
.36	.49995	-68.85	.91	.48049	129.49
.37	.49994	-70.88	.92	.47456	123.50
.38	.49993	-72.93	.93	.46680	117.14
.39	.49992	-74.99	.94	.45667	110.39
.40	.49991	-77.05	.95	.44358	103.24
.41	.49989	-79.14	.96	.42689	95.70
.42	.49988	-81.23	.97	.40609	87.83
.43	.49986	-83.34	.98	.38095	79.73
.44	.49984	-85.46	.99	.35176	71.57
.45	.49982	-87.60	1.00	.31944	63.54
.46	.49980	-89.76	1.01	.28541	55.71
.47	.49978	-91.93	1.02	.25135	48.60
.48	.49976	-94.11	1.03	.21882	42.20
.49	.49973	-96.32	1.04	.18894	36.58
.50	.49970	-98.54	1.05	.16232	31.74
.51	.49967	-100.79	1.06	.13916	27.65
.52	.49964	-103.05	1.07	.11929	24.22
.53	.49961	-105.33	1.08	.10239	21.35
.54	.49958	-107.64	1.09	.08806	18.93
.55	.49954	-109.97	1.10	.07590	16.86
.56	.49950	-112.32	1.11	.06554	15.07
.57	.49947	-114.70	1.12	.05666	13.48
.58	.49943	-117.10	1.13	.04899	12.02
.59	.49939	-119.54	1.14	.04232	10.67
.60	.49935	-122.00	1.15	.03648	9.39
.61	.49931	-124.49	1.16	.03133	8.16
.62	.49927	-127.01	1.17	.02677	6.96
.63	.49924	-129.56	1.18	.02270	5.78
.64	.49920	-132.16	1.19	.01906	4.62

C C VTF(T(S)-PHASE) VS. FREQUENCY, OUTPUT DATA, TYPE 7 FILTER, FIG.45

W1	T(JW)	O1	W1	T(JW)	O1
.10	.50000	-18.89	.65	.49984	-136.54
.11	.50000	-20.78	.66	.49982	-139.22
.12	.50000	-22.68	.67	.49979	-141.94
.14	.50000	-26.49	.69	.49971	-147.51
.15	.50000	-28.40	.70	.49965	-150.20
.17	.50000	-32.24	.72	.49949	-156.06
.19	.50000	-36.08	.74	.49924	-162.14
.20	.50000	-38.01	.75	.49907	-165.27
.22	.50000	-41.89	.77	.49859	188.09
.23	.50000	-43.84	.78	.49827	184.73
.24	.50000	-45.79	.79	.49788	181.29
.25	.50000	-47.75	.80	.49740	177.76
.26	.50000	-49.72	.81	.49682	174.13
.27	.50000	-51.69	.82	.49610	170.40
.28	.50000	-53.67	.83	.49524	166.55
.29	.50000	-55.81	.84	.49418	162.57
.30	.50000	-57.81	.85	.49288	158.45
.31	.49999	-59.81	.86	.49128	154.17
.32	.49999	-61.83	.87	.48931	149.73
.33	.49999	-63.85	.88	.48685	145.11
.34	.49999	-65.88	.89	.48378	140.27
.35	.49999	-67.92	.90	.47991	135.21
.36	.49999	-69.97	.91	.47500	129.89
.37	.49999	-72.03	.92	.46877	124.30
.38	.49998	-74.10	.93	.46086	118.43
.39	.49998	-76.19	.94	.45084	112.25
.40	.49998	-78.28	.95	.43826	105.77
.41	.49998	-80.39	.96	.42270	99.03
.42	.49997	-82.51	.97	.40386	92.06
.43	.49997	-84.65	.98	.38165	84.96
.44	.49997	-86.80	.99	.35636	77.86
.45	.49996	-88.96	1.00	.32862	70.72
.46	.49996	-91.14	1.01	.29939	64.02
.47	.49996	-93.34	1.02	.26978	57.74
.48	.49995	-95.55	1.03	.24088	51.99
.49	.49995	-97.78	1.04	.21358	46.84
.50	.49995	-100.03	1.05	.18850	42.33
.51	.49994	-102.29	1.06	.16596	38.45
.52	.49994	-104.58	1.07	.14603	35.16
.53	.49993	-106.88	1.08	.12864	32.43
.54	.49993	-109.21	1.09	.11358	30.17
.55	.49993	-111.56	1.10	.10059	28.33
.56	.49992	-113.93	1.11	.08942	26.82
.57	.49992	-116.33	1.12	.07980	25.58
.58	.49991	-118.76	1.13	.07148	24.55
.59	.49991	-121.21	1.14	.06427	23.68
.60	.49990	-123.68	1.15	.05797	22.90
.61	.49989	-126.19	1.16	.05243	22.20
.62	.49988	-128.73	1.17	.04754	21.53
.63	.49987	-131.30	1.18	.04318	20.88
.64	.49986	-133.90	1.19	.03926	20.23

APPENDIX I

Complementary Filter ADPI Coefficient Symmetry. It can be noted that complementary filter ADPI's exhibit coefficient symmetry in their numerator and denominator polynomials. This observation can be verified mathematically.

A polynomial P(s) of degree n has coefficient symmetry if

$$s^n P(1/s) = P(s) \tag{50}$$

Let $Z(s)=P(s)/Q(s)$ be the driving point impedance of the low-pass filter of the complementary filter and let deg $P=m$, deg $Q=m-1$. Then the high-pass filter has a driving point impedance $Z(1/s)$ because the low-pass to high-pass transformation is s replaced by $1/s$.

The driving point impedance of the complementary filter is

$$Z_{CF}(s) = \frac{Z(s) \;\; Z(1/s)}{Z(s) + Z(1/s)} \tag{51}$$

$$Z_{CF}(s) = \frac{P(s) \;\; P(1/s)}{P(s) \;\; Q(1/s) \; + \; P(1/s) \;\; Q(s)} \tag{52}$$

$$Z_{CF}(s) = \frac{P(s) \;\; s^m P(1/s)}{s^m \; P(s) \;\; Q(1/s) \; + s^m \; P(1/s) \;\; Q(s)} \tag{53}$$

The test for coefficient symmetry can now be applied to the numerator first and then to the denominator of $Z_{CF}(s)$.

The numerator degree is 2m and since

$$s^{2m}\left[P(s)s^{m}P(1/s)\right]\Big|_{s\to 1/s} = s^{m}P(1/s)P(s) \qquad (54)$$

the numerator polynomial is symmetric. The degree of the denominator is equal to 2m. Hence

$$\begin{aligned} & s^{2m}\left[s^{m}P(s)Q(1/s) + s^{m}P(1/s)Q(s)\right]\Big|_{s\to 1/s} = \\ & s^{2m}\left[s^{-m}P(1/s)Q(s) + s^{-m}P(s)Q(1/s)\right] = \qquad (55) \\ & s^{m}P(s)Q(1/s) + s^{m}P(1/s)Q(s) \end{aligned}$$

implies that the denominator polynomial is symmetric.

Note that the numerator and denominator polynomials degrees are both equal to 2m.

APPENDIX J

FORTRAN Program for Coefficient Evaluation of the Product of Two Polynomials with Algebraic Coefficients. This computer program is a modification of one written by Dr. Benton Weathers. Since this program is quite long and rather complicated, a self-explanatory program was added instead of a block diagram. The idea behind the program is to set up two three-dimensional matrices with each matrix representing one of the polynomials. Since each term of the polynomial has the form, $8a^3b^3s^2$, each dimension of the matrix represents an exponent of a, b, or s. And the numerical coefficient was assigned to the location given by the exponents. Having established this, the multiplying and term collecting operations are next.

```
C     PRC. FOR FINDING BIN. COFFF. OF PRODUCT POLY.-MODIFIED WEATHERS&FOWLER
C     TYPICAL POLY. TO BE MULTIPLIED ARE -
C     Z(1)=1.+2.*A*S+4.*A*S**2+(4.*A**2+2.*A**2*B)*S**3
C     +(2.*A**2+2.*A**2*B+A**3)*S**4+(4.*A**3*B+2.*A**3*B**2)*S**5
C     +2.*A**3*B*S**6+2.*A**4*B**2*S**7
C     Z(2)=2.*A**4*B**2+2.*A**3*B*S+(4.*A**3*B+2.*A**3*B**2)*S**2
C     +(2.*A**2+2.*A**2*B+A**3)*S**3+(2.*A**2+4.*A**2*B)*S**4
C     +4.*A*S**5+2.*A*S**6+S**7
C     I = NUMBER OF ALL TERMS IN ALPHANUMERIC COEFF. OF S OF ONE POLY. WHICH
C     ARE NOT ZERO.  I = 12 IN EXAMPLE.
C     J = NUMBER OF ALL TERMS IN ALPHANUMERIC COFFF. OF S OF OTHER POLY. WHICH
C     ARE NOT ZERO. J = 12
C     N1 IS LARGEST EXPONENT + ONE OF A - TERMS. N1 = 5
C     N2 IS LARGEST EXPONENT + ONE OF B - TERMS. N2 = 5
C     N3 IS LARGEST EXPONENT + ONE OF S IN BOTH POLY. N3 = 8
C     N4 = N1 + N2 - 1 AND N5 = N4**2    N4 = 9    N5 = 81
C     L1 = EXPONENT OF A-TERM, L2 = EXPONENT OF B-TERM, L3 = EXP. OF S-TERM
C     L1 = 1   L2 = 0   L3 = 1
C     M(L1,L2,L3) = NUMERICAL TERM OF ALPHANUMERIC COEFF. M(L1,L2,L3) = 2
C     L4,L6,L7,AND L9 ARE JUST DUMMY VARIABLES
C     L5 AND L8 REPRESENT THE EXP. OF S UNDER CONSIDERATION IN THE PRODUCT POLY.
C     TYPICAL DATA -
C     READ 1,I,J
C       12          12
C     READ 1,N1,N2,N3,N4
C        5           5          8          9
C     READ 1,N5
C       81
C     READ 1,L1,L2,L3
C     READ 1,M(L1+1,L2+1,L3+1)
C        0           0          0
C        1
C        1           0          1
C        2
C        1           0          2
C        4
C        2           0          3
C        4
C                    ETC.
```

```
C     READ 1,L1,L2,L3
C     READ 1,N(L1+1,L2+1,L3+1)
C        4          2          0
C        2
C        3          1          1
C        2
C        3          1          2
C        4
C        3          2          2
C        2
                         ETC.
C     DIMENSION M(N1,N2,N3),N(N1,N2,N3),K(N4,N4),K2(N5),K3(N5),K4(N5)
   22 READ 1,I,J
  226 READ 1,N1,N2,N3,N4
      READ 1,N5
      DO 2 L1=1,N1
      DO 2 L2=1,N2
      DO 2 L3=1,N3
      M(L1,L2,L3)=0
    2 N(L1,L2,L3)=0
C     THESE DO LOOPS ZERO THE MATRICES WHICH REPRESENT EACH POLY.
      DO 3 L4=1,I
      READ 1,L1,L2,L3
    3 READ 1,M(L1+1,L2+1,L3+1)
C     THE + 1 TAKES CARE OF ZERO SUBSCRIPTS
      DO 6 L4=1,J
      READ 1,L1,L2,L3
    6 READ 1,N(L1+1,L2+1,L3+1)
C     THESE DO LOOPS READ IN THE TWO POLY. INTO THEIR RESPECTIVE MATRICES
      DO 10 L5=1,2*N3-1
C     THIS DO LOOP TO STATE.10 EVALUATES THE ALGEBRAIC COEFF. FOR EACH POWER OF S
      DO 7 L4=1,2*N1-1
      DO 7 L6=1,2*N2-1
    7 K(L4,L6)=0
C     THESE DO LOOPS ZERO THE MATRIX WHICH WILL STORE THE ALG. COEFF. FOR EACH
C     POWER OF S.
      L8=L5-1
C     L8 IS THE EXP. OF S UNDER CONSIDERATION IN THE PRODUCT POLY.
      PUNCH 21,L8
```

```
      DO 12 L4=1,N3
      DO 12 L6=1,N3
      IF(L4+L6-L5-1)12,11,12
C     THIS DECISION STATE.DETERMINES CORRECT COMBINATION OF POLY. EXP.TO GIVE L8
   11 DO 12  L1=1,N1
      DO 12  L2=1.N2
      IF(M(L1,L2,L4))115,12 ,115
C     THIS IF STATE. ALLOWS DISREGARD OF MULTIPLYING WITH ZERO COEFF.
  115 DO 12 L7=1.N1
      DO 12 L9=1,N2
      J1=L1+L7-1
      J2=L2+L9-1
      K(J1.J2)=K(J1,J2)+M(L1.L2.L4)*N(L7,L9.L6)
   12 CONTINUE
C     THESE DO LOOPS CALCULATE AND STORE NUMERICAL COEFF. FOR APPROPRIATE
C     EXPONENTS OF A AND B.
      K1=0
      DO 13 I1=1,2*N1-1
C     THIS DO LOOP PUNCHES OUT EXPONENTS OF A.
      DO 13 J3=1,2*N2-1
C     THIS DO LOOP PUNCHES OUT EXPONENTS OF B.
      IF(K(I1,J3))14,13.14
C     THIS IF STATEMENT ELIMINATES PUNCHING OUT ZERO COEFF.
   14 K1=K1+1
      K2(K1)=I1-1
      K3(K1)=J3-1
      K4(K1)=K(I1,J3)
   13 CONTINUE
      IF(K1)10,10.135
C     THIS IF STATE. ELIMINATES CHECK STOP IF ALL COEFF. ARE ZERO
  135 PUNCH 20,(K4(I2),K2(I2),K3(I2),I2=1,K1)
   10 CONTINUE
    1 FORMAT(4I10)
   21 FORMAT(/27HBINOMIAL COEFFICIENT OF S**I3)
   20 FORMAT(I10,I3.1H.I2,5X,I10,I3,1H,I2,5X,I10,I3.1H.I2)
      END
```

```
BINOMIAL COEFFICIENT OF S**  0
         2   4, 2

BINOMIAL COEFFICIENT OF S**  1
         2   3, 1                 4   5, 2

BINOMIAL COEFFICIENT OF S**  2
         4   3, 1                 2   3, 2                  4   4, 1
         8   5, 2

BINOMIAL COEFFICIENT OF S**  3
         2   2, 0                 2   2, 1                  1   3, 0
        16   4, 1                 4   4, 2                  8   6, 2
         4   6, 3

BINOMIAL COEFFICIENT OF S**  4
         2   2, 0                 4   2, 1                  4   3, 0
         4   3, 1                 2   4, 0                 16   4, 1
         8   4, 2                 8   5, 1                  4   5, 2
         4   6, 2                 4   6, 3                  2   7, 2

BINOMIAL COEFFICIENT OF S**  5
         4   1, 0                12   3, 0                 16   3, 1
         4   4, 0                20   5, 1                 20   5, 2
         4   5, 3                 2   6, 1                  8   7, 3
         4   7, 4

BINOMIAL COEFFICIENT OF S**  6
         2   1, 0                 8   2, 0                  8   3, 0
        16   3, 1                 8   4, 0                 12   4, 1
         4   4, 2                 4   5, 0                 10   5, 1
        12   5, 2                 4   5, 3                  4   6, 1
        10   6, 2                 4   6, 3                  4   7, 3

BINOMIAL COEFFICIENT OF S**  7
         1   0, 0                20   2, 0                 12   4, 0
        28   4, 1                12   4, 2                  4   5, 0
         4   5, 1                 1   6, 0                 20   6, 2
        16   6, 3                 4   6, 4                  4   8, 4

BINOMIAL COEFFICIENT OF S**  8
         2   1, 0                 8   2, 0                 16   3, 0
         8   3, 1                 4   4, 0                 12   4, 1
         8   4, 2                 2   5, 0                 12   5, 1
        12   5, 2                 4   5, 3                  4   6, 1
        10   6, 2                 4   6, 3                  4   7, 3

BINOMIAL COEFFICIENT OF S**  9
         4   1, 0                16   3, 0                 12   3, 1
         4   4, 0                12   5, 1                 24   5, 2
         8   5, 3                 2   6, 1                  8   7, 3
         4   7, 4
```

BINOMIAL COEFFICIENT OF S** 10

4 2, 0	2 2, 1	4 3, 0
4 3, 1	2 4, 0	16 4, 1
8 4, 2	4 5, 1	8 5, 2
4 6, 2	4 6, 3	2 7, 2

BINOMIAL COEFFICIENT OF S** 11

2 2, 0	2 2, 1	1 3, 0
16 4, 1	4 4, 2	4 6, 2
8 6, 3		

BINOMIAL COEFFICIENT OF S** 12

4 3, 1	2 3, 2	4 4, 1
8 5, 2		

BINOMIAL COEFFICIENT OF S** 13

2 3, 1	4 5, 2

BINOMIAL COEFFICIENT OF S** 14

2 4, 2

APPENDIX K

<u>ADPI Parameter Evaluation for a Type 7 Filter.</u> In applying the aidentity algorithm to the ADPI of Type 7, three equations, (23), (24), and (25), and three unknowns, C, C_1, and m, result. Equation (23) can be solved for C_1 in terms of C and m. This results in equation (56) where $a=1+m$. Equation (24) can be manipulated to give equation

$$C_1 = \frac{aC}{a^2C^2 + aC - 1} \tag{56}$$

(57) and if the expression for C_1 in equation (56) is substituted for the C_1 in equation (57), the expression for C given in equation (58) will be obtained.

$$C_1(a^3bCm+a^3bC^2m+a^2C^2+a^2C-a^2bC-a^2b-aCm-1) + a^3bC^2 - a^3bC - aC = 0 \tag{57}$$

$$C = \sqrt{3 - m^2/(1+m)^2} \tag{58}$$

Equation (25) can be rearranged to give equation (59). Substituting the expression for C_1 from equation (56) into equation (59) results in equation (61) after much expanding, manipulating, consolidating, and factoring. Now since equation (56) has been used with both (57) and (59) to give (58) and (61), these two will now be combined to give a polynomial with only one parameter, m. Using equation (58) C^4, C^3, and C^2 can be generated. Substituting these into

$$C_1^{\,2}X + C_1Y + aCZ = 0 \qquad (59)$$

where

$$X = C^2a^2b + 2Ca^2b - Cabm + a^2b - abm - b$$

$$Y = C^2a^4b^2 + C^2a^3 + Ca^4b^2 + Ca^2m^2 - ab - a^2b - ab - a - 1 \qquad (60)$$

$$Z = Ca^3b + Ca - a^2b - ab - a - 1$$

$$AC^4 + BC^3 + DC^2 + EC + F = 0 \qquad (61)$$

where

$$A = (2+m-m^2-m^3)(1+m)^4$$

$$B = (2+m-2m^2-m^3+m^4)(1+m)^3$$

$$D = (-7-3m+2m^2+2m^3+2m^4+m^5)(1+m)^2 \qquad (62)$$

$$E = (-4-2m+2m^2+m^3+m^5)(1+m)$$

$$F = 6+2m-3m^2-m^3-m^4-m^5$$

equation (61) and clearing denominators results in equation (63) after factoring and consolidating. Equation (63) can not be factored for roots of m. Therefore notice that equation

$$(3+2m-11m^2-7m^3+11m^4+7m^5-3m^6-2m^7) + (2+m-6m^2-3m^3+5m^4+2m^5-m^6)\sqrt{3-m^2} = 0 \qquad (63)$$

(63) has the form $P+Q\sqrt{R}=0$. If this is multiplied by the factor, $(P-Q\sqrt{R})$, the result will be $P^2-Q^2R=0$ which can be factored; however, this polynomial now has in it spurious roots of the factor, $(P-Q\sqrt{R})$. So any roots obtained will have to be checked in equation (63) to make sure that they

are not just roots of the factor, $(P-Q\sqrt{R})$. Hofman's computer program of Appendix M and N using BAIRSTOW'S method on the IBM 1401-1410 gave the roots to the fourteenth order polynomial. These roots are given in equation (64) and only one was realizable and not trivial. Root q_{14} is the only realizable non-trivial root and when substituted into equation (63) is found to be a root. With this value of m substituted into equation (58), C is found to be a realizable number when

$$P^2 - Q^2R = \prod_{n=1}^{14} (m+q_n) \tag{64}$$

where

$$\begin{aligned}
q_1 &= 1 & q_8 &= 0.88 - j0.49 \\
q_2 &= 1 & q_9 &= 0.88 + j0.49 \\
q_3 &= -1 & q_{10} &= -1.59 \\
q_4 &= -1 & q_{11} &= -0.245 - j0.515 \\
q_5 &= 0.7264 & q_{12} &= -0.245 + j0.515 \\
q_6 &= 1.7 & q_{13} &= -1.594 \\
q_7 &= 1.25 & q_{14} &= -0.7143309
\end{aligned} \tag{65}$$

evaluated and then C_1 is evaluated from equation (56). This gives

$$\begin{aligned}
m &= 0.7143309 \\
C &= 0.92041 \\
C_1 &= 0.51437
\end{aligned} \tag{66}$$

APPENDIX L

<u>FORTRAN Program for Evaluation of Type 7 Filter ADPI Coefficients.</u> With the parameter values of equation (66), the coefficients expressed algebraically in equations (22) were evaluated with the computer program given on pages 126 and 127 with the results given on page 127.

```
MON$$     JOB  FILTER COEFFICIENTS CASE 7              23 DEC 64     J
MON$$     COMT 15 MIN, 10 PAGES, FOWLER, EE
MON$$     ASGN MJB,12
MON$$     ASGN MGO,16
MON$$     MODE GO,TEST
MON$$     EXEQ FORTRAN,,,,,,,FANOUT
      DIMENSION C(10),D(10)
    9 FORMAT(5X,10H A( 0)  = ,F10.5,5X,10H B( 0)  = ,F10.5)
    2 FORMAT(5X,3H A(,I2,5H)  = ,F10.5,5X,3H B(,I2,5H)  = ,F10.5)
      FM=0.7143309
      FK1=0.92041056
      FK2=0.51436894
      A=1.+FM
      B=1.-FM
      AB=A*B
      A2B=AB*A
      A3B=A2B*A
      A4B=A3B*A
      A5B=A4B*A
      A6B2=A5B*A*B
      A2=A*A
      A3=A2*A
      A4=A3*A
      BK2=B*FK2
      FK1K2=FK1*FK2
      FK22=FK2*FK2
      FK11=FK1*FK1
      F1=1.+FK1
      F2=1.+FK1*FM
      F3=1.+BK2
      F4=1.+FK2*FM
      C0=A3B*FK1K2
      C(1)=A4B*FK1K2*F1+A*A*FK1K2
      C(2)=A4B*FK1*(FK1+FK2*F2)+  A3*FK1K2*F1+  A2*FK2*(FK1+BK2*F1)
      C1=A5B*FK1*(FK1+BK2*F1)+A3*FK1*(FK1+FK2*F2)+A3*FK2*F1*(FK1+BK2*F1)
      C(3)=C1+A*FK2*(FK1+FK2*F2)
      C2=A5B*FK11+A4*FK1*(FK1+BK2*F1)+A3*(FK1+BK2*F1)*(FK1+FK2*F2)
      C(4)=C2+A2*FK2*F1*(FK1+FK2*F2)+A*FK22*F1
      C3=A6B2*FK11+A4*FK11+A4*(FK1+BK2*F1)**2+A2*(FK1+FK2*F2)**2+FK22
      C(5)=C3+A2*FK22*F1**2
      C(6)=C(4)
      C(7)=C(3)
      C(8)=C(2)
      C(9)=C(1)
      C(10)=C0
      D0=C0
      D(1)=A3B*FK1*F4+A2*FK1K2+A2*BK2
      D(2)=A4B*FK1*F3+A2*FK1*F4+A2*FK2*(FK1+BK2*F1)+A3B*FK2*F1+A*FK2
      D1=A4B*FK1+A3*FK1*F3+A2*F4*(FK1+BK2*F1)+A*FK2*(FK1+FK2*F2)
      D(3)=D1+A3B*(FK1+FK2*F2)+A2*FK2*F1+A*FK2*F3
      D2=A5B*B*FK1+A3*FK1+A3*F3*(FK1+BK2*F1)+A*F4*(FK1+FK2*F2)
      D3=A*FK22*F1+A4B*(FK1+BK2*F1)+A2*(FK1+FK2*F2)+A2*FK2*F1*F3
      D(4)=D2+D3+FK2*F4
      D4=A4B*FK1+A3*(FK1+BK2*F1)+A2*F3*(FK1+FK2*F2)+A*FK2*F1*F4+FK22
```

```
      D(5)=2.*D4
      D(6)=D(4)
      D(7)=D(3)
      D(8)=D(2)
      D(9)=D(1)
      D(10)=D0
      WRITE (2,9)C0,D0
      DO 3 I=1,10
      WRITE(2,2)I,C(I),I,D(I)
    3 CONTINUE
      STOP
      END
     MON$$      EXEQ LINKLOAD
                CALL FANOUT
     MON$$      EXEQ FANOUT,MJB
```

```
1     MON$$ SCS JOB  FILTER COEFFICIENTS CASE 7              23 DEC 64    J
      A( 0)   =       .68140       B( 0)   =       .68140
      A( 1)   =      3.63470       B( 1)   =      3.63470
      A( 2)   =     10.42514       B( 2)   =     10.42514
      A( 3)   =     20.45234       B( 3)   =     20.45234
      A( 4)   =     29.90444       B( 4)   =     29.93704
      A( 5)   =     33.93411       B( 5)   =     33.77306
      A( 6)   =     29.90444       B( 6)   =     29.93704
      A( 7)   =     20.45234       B( 7)   =     20.45234
      A( 8)   =     10.42514       B( 8)   =     10.42514
      A( 9)   =      3.63470       B( 9)   =      3.63470
      A(10)   =       .68140       B(10)   =       .68140
```

APPENDIX M

Bairstow's Root Extraction. Mr. Larry Hofman prepared the following analysis of root extraction using Bairstow's method and Hamming's (12) suggested numerical methods. However Hofman added procedures to insure convergence for most polynomials.

Bairstow's Method. Let the polynomial to be factored be

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0 \quad (67)$$

There is a quadratic factor of the form $Ax^2 + Bx + C$. Assume $a_n = 1$ and guess at the factor

$$x^2 + px + q \quad (68)$$

Divide the polynomial by the quadratic factor and obtain a quotient and a remainder, e.g.,

$$a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0 = (x^2 + px + q)(b_n x^{n-2} + b_{n-1} x^{n-3} + \dots + b_2) + b_1 x + b_0 \quad (69)$$

The peculiar subscripts on the b's make notation easier. In a skeleton synthetic form

$$\begin{array}{c|cccccccc}
1 \quad p \quad q & a_n & a_{n-1} & a_{n-2} & a_{n-3} & \cdots & a_2 & a_1 & a_0 \\
 & — & — & qb_n & qb_{n-1} & \cdots & qb_4 & qb_3 & qb_2 \\
 & — & pb_n & pb_{n-1} & pb_{n-2} & \cdots & pb_3 & pb_2 & — \\
\hline
 & b_n & b_{n-1} & b_{n-2} & b_{n-3} & \cdots & b_2 & \boxed{b_1 \quad b_0} &
\end{array}$$

where the remainder is

$$b_1 x + b_0$$

The algebraic relations between the coefficients are

$$\begin{aligned}
b_n &= a_n \\
b_{n-1} &= a_{n-1} - pb_n \\
b_{n-2} &= a_{n-2} - pb_{n-1} - qb_n \\
&\ldots\ldots\ldots\ldots\ldots \qquad (70) \\
b_{n-k} &= a_{n-k} - pb_{n-k+1} - qp_{n-k+2} (k=2,\ 3,\ \ldots,\ n-1) \\
&\ldots\ldots\ldots\ldots\ldots \\
b_0 &= a_0 - qb_2
\end{aligned}$$

The desired quadratic factor is obtained if, and only if, the remainder is identically zero. That is,

$$b_1 = b_0 = 0$$

Now find some exact method of correcting the guess of B and C in order that the above conditions are met. Consider the coefficients b_1 and b_0 to be functions of p and q:

$$\begin{aligned}
b_1 &= b_1(p,q) \\
b_0 &= b_0(p,q)
\end{aligned}$$

Using Newton's method in two dimensions, expand b_1 and b_0 about the present guess (p and q). Writing B and C as the desired solution,

$$b_1(B,C) = 0 = b_1(p,q)+\frac{\partial b_1}{\partial p}\Delta p+\frac{\partial b_1}{\partial q}\Delta q+ \ \dots \qquad (71)$$

$$b_0(B,C) = 0 = b_0(p,q)+\frac{\partial b_0}{\partial p}\Delta p+\frac{\partial b_0}{\partial q}\Delta q+ \ \dots$$

where

$$\Delta p = B - p \qquad (72)$$

$$\Delta q = C - q$$

are the errors to be corrected (approximately) for the next guess. Neglecting all but the linear terms in (71), results in a pair of linear equations for the changes to be made in p and q. The problem is to find the partial derivatives which are the coefficients of the unknowns Δp and Δq. Differentiating equations (70) with respect to p, gives

$$\frac{\partial b_n}{\partial p} = 0$$

$$\frac{\partial b_{n-1}}{\partial p} = -b_n - p\frac{\partial b_n}{\partial p}$$

$$\frac{\partial b_{n-2}}{\partial p} = -b_{n-1} - p\frac{\partial b_{n-1}}{\partial p} - q\frac{\partial b_n}{\partial p}$$

$$\cdots\cdots\cdots\cdots$$

$$\frac{\partial b_{n-k}}{\partial p} = -b_{n-k+1} - p\frac{\partial b_{n-k+1}}{\partial p} - q\frac{\partial b_{n-k+2}}{\partial p}$$

$$\cdots\cdots\cdots\cdots$$

$$\frac{\partial b_0}{\partial p} = \qquad - q\frac{\partial b_2}{\partial p}$$

Now write
$$\frac{\partial b_k}{\partial p} = - c_k^* \tag{73}$$

then

$$\begin{aligned}
c_n^* &= 0 \\
c_{n-1}^* &= b_n \quad -pc_n^* \\
c_{n-2}^* &= b_{n-1} \quad -pc_{n-1}^* \quad - qc_n^* \\
&\dots\dots\dots\dots \\
c_{n-k}^* &= b_{n-k+1} - pc_{n-k+1}^* - qc_{n-k+2}^* \\
&\dots\dots\dots\dots \\
c_0^* &= \qquad\qquad\qquad - qc_2^*.
\end{aligned} \tag{74}$$

These equations are practically in the same form as (70). This suggests repeating the process of synthetic division using the quadratic factor of $x^2 + px + q$ on the b's to obtain coefficients c_k. Then

$$\begin{array}{ccc|cccccccc}
1 & p & q & b_n & b_{n-1} & b_{n-2} & b_{n-3} & \cdots & b_2 & b_1 & b_0 \\
\hline
 & & & — & — & qc_n & qc_{n-1} & \cdots & qc_4 & qc_3 & qc_2 \\
 & & & — & pc_n & pc_{n-1} & pc_{n-2} & \cdots & pc_3 & pc_2 & — \\
\hline
 & & & c_n & c_{n-1} & c_{n-2} & c_{n-3} & \cdots & c_2 & \boxed{c_1} & \boxed{c_0.}
\end{array}$$

where

$$\begin{aligned}
c_n &= b_n \\
c_{n-1} &= b_{n-1} - pc_n \\
c_{n-2} &= b_{b-2} - pc_{n-1} \quad - qc_n \\
&\dots\dots\dots\dots
\end{aligned} \tag{75}$$

$$c_{n-k} = b_{n-k} - pc_{n-k+1} - qc_{n-k+2}$$

$$\cdots\cdots\cdots \qquad (75)$$

$$c_1 = b_1 - pc_2 - qc_3$$

$$c_0 = b_0 - qc_2$$

The partial derivatives desired in (71), using (73), are

$$\frac{\partial b_1}{\partial p} = -c_1^* \quad \frac{\partial b_0}{\partial p} = -c_0^*$$

Comparing (74) and (75) gives

$$c_{k-1}^* = c_k \qquad (k = n, n-1, \ldots , 3, 2)$$

$$c_0^* = c_1 - b_1 + pc_2$$

Hence

$$\frac{\partial b_1}{\partial p} = -c_2 \quad \frac{\partial b_0}{\partial p} = -(c_1 - b_1 + pc_2) \qquad (76)$$

Now examine the process for the partial derivatives with respect to q. Again differentiating (70) gives

$$\frac{\partial b_n}{\partial q} = 0$$

$$\frac{\partial b_{n-1}}{\partial q} = 0$$

$$\frac{\partial b_{n-2}}{\partial q} = -b_n - p\frac{\partial b_{n-1}}{\partial q} - q\frac{\partial b_n}{\partial q}$$

$$\cdots\cdots\cdots$$

$$\frac{\partial b_{n-k}}{\partial q} = -b_{n-k+2} - p\frac{\partial b_{n-k+1}}{\partial q} - q\frac{\partial b_{n-k+2}}{\partial q}$$

. . . .

$$\frac{\partial b_0}{\partial q} = -b_2 \qquad - q\frac{\partial b_2}{\partial q}$$

Now set

$$\frac{\partial b_k}{\partial q} = -c_k^{**} \tag{77}$$

to get

$$
\begin{aligned}
c_n^{**} &= 0\\
c_{n-1}^{**} &= 0\\
c_{n-2}^{**} &= b_n \qquad - pc_{n-1}^{**} - qc_n^{**}\\
&\ldots\ldots\ldots\ldots \\
c_{n-k}^{**} &= b_{n-k+2} - pc_{n-k+1}^{**} - qc_{n-k+2}^{**}\\
&\ldots\ldots\ldots\ldots \\
c_0^{**} &= b_2 \qquad - qc_2^{**}
\end{aligned}
\tag{78}
$$

Since $c_n^{**} = c_{n-1}^{**} = 0$, it is necessary to identify

$$c_{k-2}^{**} = c_k \qquad (k = n, n-1, \ldots, 4,3)$$

$$c_0^{**} = c_2 + pc_3$$

if (75) and (78) are to be compared.

The partial derivatives desired for (71) are

$$\frac{\partial b_1}{\partial q} = -c_1^{**} = -c_3 \qquad \frac{\partial b_0}{\partial q} = -c_0^{**} = -(c_2 + pc_3).$$

Thus $$b_1(p,q) = c_2 \Delta p + c_3 \Delta q \tag{79}$$

$$b_0(p,q) = (c_1 - b_1 + pc_2)\Delta p + (c_2 + pc_3)\Delta q$$

Solving equation (79) simultaneously, results in

$$\Delta p = \frac{b_1(c_2 + pc_3) - c_3 b_0}{c_2^{\,2} + c_3(b_1 - c_1)} \tag{80}$$

$$\Delta q = \frac{b_0 c_2 - b_1(c_1 - b_1 + pc_2)}{c_2^{\,2} + c_3(b_1 - c_1)}$$

Now replace the value of p and q with

$$p \longrightarrow p + \Delta p$$

$$q \longrightarrow q + \Delta q$$

and repeat the above process until the values of b_1 and b_0 are sufficiently small. The convergence, when it works, is quadratic; that is, the errors, when small, are approximately squared each step. Thus an iterative process is obtained which will converge upon a quadratic factor of the original polynomial. When found, it can be factored out (the first division step) and then use the quotient as a new polynomial to be examined by the same process.

The errors introduced by this method are accumulated as each factor is removed (division by an inexact root and then discarding the remainder). Thus a polynomial of large order will require a high degree of accuracy in order that the last factors will have an acceptable accuracy also.

The convergence of Bairstow's method is somewhat sensitive to the initial guess of p and q. Since no value has been found for p and q which will cause convergence for all cases, Hofman devised a program that tries a succession of initial guesses, as outlined in Table I, in an attempt to extract a stubborn factor. This program has yet to fail after many trials.

trial	p	q	roots	
1	4	3	-1	-3
2	2	1	-1	-1
3	0	-1	-1	+1
4	2	2	$-1 \pm j1$	
5	-2	2	$+1 \pm j1$	

Table I

APPENDIX N

Hofman's FORTRAN Program for Polynomial Root Extraction. With the analysis for root extraction as presented in Appendix M, Hofman wrote the following FORTRAN computer program for use on the IBM 1401-1410 digital computer.

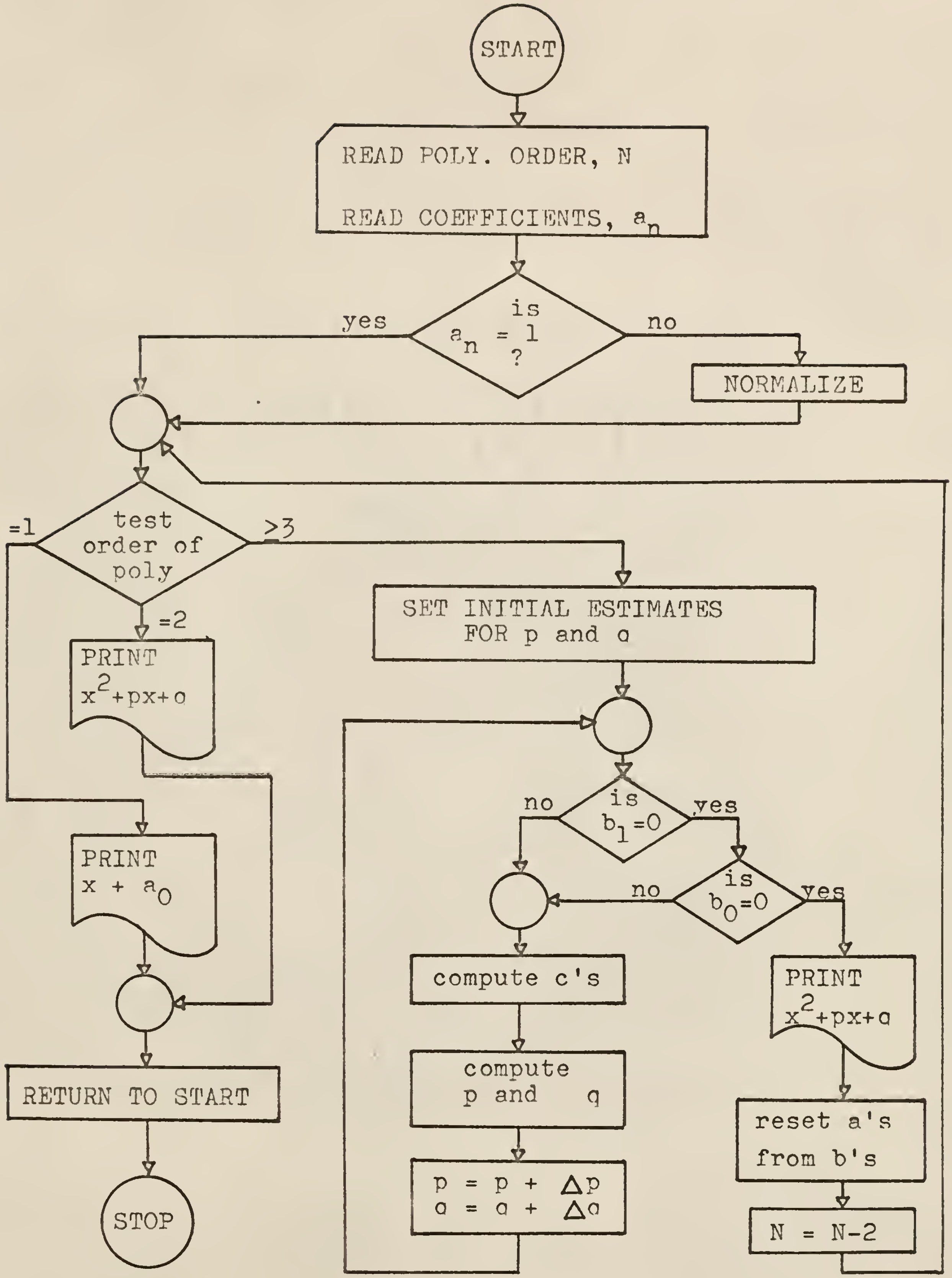

Fig. 59. Block diagram of Bairstow's polynomial factoring computer program.

```
      MON$$      JOB  BAIRSTOW METHOD OF POLYNOMIAL FACTOR  1/1/64    LBHJ
      MON$$      COMT 5 MIN, 5 PAGES, HOFMAN, ELECT ENGG
      MON$$      ASGN MJB,12
      MON$$      ASGN MGO,16
      MON$$      MODE GO
      MON$$      EXEQ FORTRAN,,,18,3,,,BAIRSTOW
       DIMENSION A(20),B(20),C(20),PI(5),QI(5)
       EQUIVALENCE (A1,A(1)),(B1,B(1)),(B2,B(2)),(B3,B(3)),(C1,C(1)),
      1(C2,C(2)),(C3,C(3)),(C4,C(4))
1      FORMAT(I2)
2      FORMAT(E14.8)
3      FORMAT(18HKMETHOD HAS FAILED)
4      FORMAT(1X,F16.8)
5      FORMAT(4HSX =,F11.7,25H ,                        X =,F11.7)
6      FORMAT(4HSX =,F11.7,4H +J ,F11.7,3X,7H ,   X =,F11.7,4H - J,F11.7)
7      FORMAT(4HSX =,F11.7)
8      FORMAT(39H1COEFFICIENTS IN DECREASING POWERS OF X/)
9      FORMAT(1HK,60X,61HITERATION    B(1)            B(0)                P
      1              Q/)
10     FORMAT(61X,I5,3X,1PE14.7,3(1PE15.7))
C      SET 5 INITIAL GUESSES.
       PI(1)=4.
       QI(1)=3.
       PI(2)=2.
       QI(2)=1.
       PI(3)=0.
       QI(3)=-1.
       PI(4)=2.
       QI(4)=2.
       PI(5)=-2.
       QI(5)=2.
C      READ ORDER N.  N=0 TO TERMINATE PROGRAM.
12     READ(1,1)N
       IF(N.EQ.0) STOP
       N=N+1
       WRITE(3,8)
       DO 14 I=1,N
       J=N+1-I
C      READ COEFFICIENTS IN DECREASING POWERS.
       READ(1,2)A(J)
14     WRITE(3,4)A(J)
       IF(A(N).EQ.1.) GO TO 16
       DO 15 I=1,N
C      NORMALIZE COEFFICIENTS.
15     A(I)=A(I)/A(N)
16     I=1
       WRITE(3,9)
17     IF(N-3) 130,120,18
18     L=N-2
       ITRY=1
C      SET INITIAL GUESS FOR P AND Q.
20     P=PI(ITRY)
       Q=QI(ITRY)
       ITCNT=1
C      CALCULATE BS.
```

```
25    B(N)=1.
      B(N-1)=A(N-1)-P
      DO 30 K=2,L
      M=N-K
30    B(M)=A(M)-P*B(M+1)-Q*B(M+2)
      B1=A1-Q*B3
      WRITE(3,10)ITCNT,B2,B1,P,Q
C     CHECK ACCURACY OF GUESS.
      IF(ABS(B2).GE..00000001) GO TO 45
      IF(ABS(B1).LT..00000001) GO TO 60
45    ITCNT=ITCNT+1
      IF(ITCNT.GT.25) GO TO 150
C     CALCULATE CS FOR CORRECTION OF P AND Q.
      C(N)=1.
      C(N-1)=B(N-1)-P
      DO 50 K=2,L
      M=N-K
50    C(M)=B(M)-P*C(M+1)-Q*C(M+2)
      C1=B1-Q*C3
      DENOM=C3*C3+C4*(B2-C2)
      IF(DENOM.EQ.0.) GO TO 55
C     CALCULATE DELTA P AND DELTA Q.
      DELTP=(B2*(C3+P*C4)-C4*B1)/DENOM
      DELTQ=(C3*B1-(B2*(C2-B2+P*C3)))/DENOM
      GO TO 57
55    DELTP=.1
      DELTQ=.1
C     CORRECT P AND Q.
57    P=P+DELTP
      Q=Q+DELTQ
      GO TO 25
C     ROUTINE TO FACTOR QUADRATIC.
60    DSCRM=P*P-4.*Q
      IF(DSCRM.LT.0.) GO TO 110
      ROOT1=(-P+SQRT(DSCRM))*.5
      ROOT2=(-P-SQRT(DSCRM))*.5
      WRITE(3,5)ROOT1,ROOT2
80    N=N-2
      DO 90 J=1,N
90    A(J)=B(J+2)
      GO TO (17,12),I
110   REAL=-P*.5
      CXPT=SQRT(-DSCRM)*.5
      WRITE(3,6)REAL,CXPT,REAL,CXPT
      GO TO 80
C     FACTOR LAST QUADRATIC REMAINING.
120   I=2
      P=A(2)
      Q=A(1)
      GO TO 60
C     REMOVE LAST LINEAR FACTOR.
130   ROOT=-A(1)
      WRITE(3,7)ROOT
      GO TO 12
```

```
C     PREPARE FOR ANOTHER INITIAL GUESS.
150   ITRY=ITRY+1
      IF(ITRY.LE.5) GO TO 20
      WRITE(3,3)
      GO TO 12
      END
    MON$$     EXEQ LINKLOAD
              CALL BAIRSTOW
    MON$$     EXEQ BAIRSTOW,MJB
```

CONSTANT-K COMPLEMENTARY FILTERS

by

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B.S.E.E., Kansas State University, 1957

AN ABSTRACT OF A MASTER'S THESIS

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Zobel, Bode, Guillemin, Norton, Rowlands, and Szentirmai used several methods with limited success to improve the driving point impedance of fan-out filter configurations. Fritzemeyer used these ideas and King's approximate identity in presenting the aidentity algorithm and impedance elision design procedures for complementary filters.

This thesis presents in detail these two design procedures and expands on the analysis of Fritzemeyer's interactance (a fan-out filter figure of merit). The Zobel process for improvement of characteristic impedance and the driving point impedance is the focus of interest.

This investigation defines, codifies, and verifies the Zobel process, aidentity algorithm, and impedance elision as methods of improving the frequency response characteristics of constant-k complementary filters. It is shown that these methods not only improve the aidentity driving point impedance or interactance, but also the voltage transfer function with linear phase shift.

Verification is shown with graphs of aidentity driving point impedance, interactance, and voltage transfer function versus frequency. These characteristics were computed on the IBM 1620 and IBM 1401-1410 digital computers using FORGO and FORTRAN languages.

In the course of this verification several related topics of interest such as the FORGO program for evaluating the coefficients of a specialized "ladder" network's VTF,

the FORTRAN program for coefficient evaluation of the product of two polynomials with algebraic coefficients, the proof of the complementary filter ADPI coefficient symmetry, and the ADPI parameter evaluation for a Type 7 filter were studied and are included in the appendices.